8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

DYNAMIC BETI/FETI-METHOD WITH NONCONFORMING INTERFACES

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Key Words: *Time domain boundary element methods, Elastodynamics, Non-conforming interfaces, FEM-BEM coupling.*

ABSTRACT

The coupling of finite and boundary element methods is well established for elliptic problems such as elastostatics even for non-conforming interfaces [5]. Similarly, the FETI method [1] and its extension to boundary element methods, the BETI method [3], have been developed. In this work, these ideas are transferred to dynamic problems, where the boundary element method is especially appealing for the treatment of unbounded domains [4].

The main idea is based on the realization of Dirichlet-to-Neumann maps for each subdomain independently by either a finite element or a boundary element discretization. Whereas such concept has been analyzed for the case of elliptic problems [5], it is now carried out for dynamic problems at each time step. In case of a finite element discretization, the system of ordinary partial differential equations is treated by the Newmark algorithm [2] and the resulting series of systems of equations has at each time step the same algebraic structure as in the static case. The Schur complement of such a system is thus a finite element realization of the discrete Dirichlet-to-Neumann map. On the other hand, the time domain boundary element method yields a system of convolution equations. Here, the convolution quadrature method is used [4] which finally gives a series of linear systems of equations which also gives rise to the discrete Dirichlet-to-Neumann map.

The interface conditions due to the partitioning of the original problem are incorporated by means of Lagrange multipliers in a weighted form. This enables the independent choice of interface discretizations as known from the mortar methods [6]. Hence, the nodes of the spatial discretizations need not be coincident and, moreover, different polynomial orders of trial functions can be used at the respective sides of the interfaces. The global system of equations is arranged in the same way as in the FETI and BETI methods [1, 3] and its solution is carried out by a parallel algorithm.

An example of the application of this methodology is the numerical analysis of an individual footing which is subject to a vertical step load on its upper surface and resting on an elastic halfspace. The discretization and the numerical outcome are shown in figure 1. Clearly, the discretization is non-conforming since neither the nodes are coincident nor the approximation orders are equal for the different subdomains. The results show the vertical displacements at different points compared with the

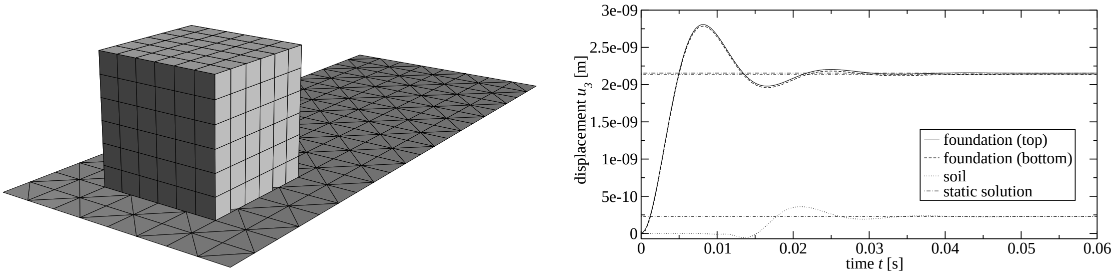

Figure 1: Numerical analysis of an individual footing under a dynamic step load — discretization by finite and boundary element methods (left) and vertical displacements against time for different positions compared with the corresponding static solution.

solution of the corresponding static problem. These points are the midpoint of the upper surface of the foundation (top), the midpoint of the lower surface (bottom), and a point on the surface of the soil at the far end of the discretization. The propagation of the disturbance along the surface is visible for the observation point on the soil surface, which consists of the pressure, shear and Rayleigh waves. Moreover, the results all converge to the static solution which indicates that the geometric damping of the halfspace is represented well and no spurious wave reflections occur.

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