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Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations

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ABSTRACT

We present a discontinuous Galerkin finite element method (DGFEM) for partial differential equations containing nonconservative products [4] as may occur e.g. in dispersed multiphase flow equations. Partial differential equations containing nonconservative products cannot be written in divergence form which causes problems once the solution becomes discontinuous, because the weak solution in the classical sense of distributions does not exist. Consequently, no Rankine-Hugoniot shock conditions can be defined. To overcome these problems, we use the theory of Dal Maso, LeFloch and Murat [1] in which a definition is given to nonconservative products in points where the solution in discontinuous. A problem with this theory, however, is the introduction of a path in phase space connecting the left and right state across a discontinuity. The construction of this path can be a very difficult and costly job. Numerical test cases to investigate the influence of the path, however, show that no relevant changes in the numerical solution occurs when the path is changed.

Within the DGFEM, the choice of the numerical flux is critical to obtain a correct solution and we also present a new numerical flux for partial differential equations containing nonconservative products. This numerical flux for NonConservative Products, or NCP flux, is such that if the nonconservative partial differential equation can be written in divergence form, it will reduce to the HLL flux [6].

The DGFEM for nonconservative partial differential equations has been extensively tested in 1 and 2 dimensional test cases for the shallow water equations, shallow water equations including sediment transport [5] and a depth-averaged two-phase flow model [3]. For the shallow water equations, these test cases included subcritical, transcritical and supercritical flow over a fixed topography, as well as more complex test cases as proposed by LeVeque [2] in which a perturbation of a steady state is considered. Hydraulic and sediment transport through a contraction was simulated using the shallow water equations coupled to a sediment equation while subcritical and supercritical flow, as well as a Riemann-type problem, were simulated using the depth-averaged two-phase flow model.

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