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DISCONTINUOUS GALERKIN METHODS FOR MORPHODYNAMIC MODELLING

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ABSTRACT

The interaction between sediment transport and water flow plays an important role in many river and coastal engineering applications. In recent years, the improved understanding of physical processes involved in the study of river hydraulics has led to the development of physical-mathematical formulations to explain the natural phenomena and to forecast changes due to, for example, human interference. Nevertheless, accurate representation of morphodynamic processes and the ability to propagate changes in the riverbed over a wide range of space and time scales make the design and implementation of appropriate numerical schemes challenging.

Prediction of changes of the bed in natural channels can be analysed by coupling a hydrodynamic flow solver which acts as a sediment driving force and a bed evolution model which accounts for sediment flux and bathymetry changes. Such a modelling system is often referred to as a *morphodynamic model*. The morphodynamic model emerges as a mixed hyperbolic–parabolic system of partial differential equations (PDE's). It is based on a depth–average over the water column resulting in shallow water theory augmented by a flow resistance term, together with a depth–averaged conservation law expressing continuity of sediment. A phenomenological sediment transport function relates the rate of sediment transport to the local mean fluid velocity. Here, we consider that sediment particles are carried along via *bedload* transport.

There are special difficulties associated with solving hyperbolic equations, including the propagation of sediment bores or discontinuous steps in the bedform, and a good numerical implementation must overcome these problems. In this work, we use a novel space and space–time discontinuous Galerkin finite element method (DGFEM). The use of DGFEM methods for these problems is also of interest because it is local and can thus deal efficiently with: (i) the improvement of the order of accuracy, thus allowing efficient p-adaptivity; (ii) the refinement of the grid, without taking into account the continuity restrictions typical of conforming finite element methods, thus allowing efficient h-adaptivity; and, (iii) performing parallel computations.

(a) (b)

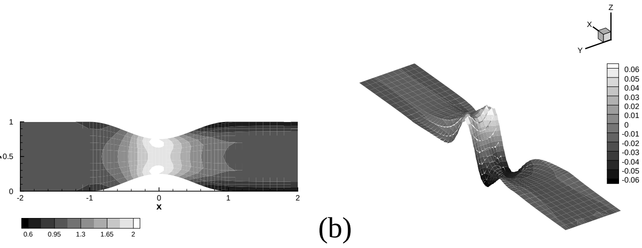

Figure 1: Flow and sediment transport in a contraction channel (a) streamwise discharge and (b) bottom profile.

For a DGFEM discretization of hydrodynamic shallow water flows, we refer to [2]. In this work, we extend and refine that method to include the bed evolution as well and a partly nonconservative formulation is used which allows the application of the unified space and space–time discontinuous Galerkin discretization for hyperbolic systems of partial differential equations with nonconservative products developed in [1] to solve the entire morphodynamic model. In our case, the nonconservative product consists of the topographic terms present in the momentum equations. For the marching in time, we have made use of advanced time stepping schemes to deal with the multiscale property of the morphodynamic problem. For space DGFEM, we intertwine a fifth-order Runge-Kutta scheme for the fast or pseudo-time to solve the hydrodynamic equations to steady state. It is designed to be a dissipative time integration scheme to efficiently reach this hydrodynamic steady–state in pseudo–time [5]. An accurate explicit time discretization is used for the sediment equation (a third order Runge-Kutta scheme [2,3]). For the space-time DGFEM, we used the fifth-order Runge-Kutta scheme in pseudo-time of Van der Vegt and Van der Ven [5].

In this work, the resulting numerical scheme is verified by comparing simulations against an extensive suite of (semi–)analytical solutions and their applicability is shown in two test cases: the evolution of an initially flat bed in a channel with a contraction (Figure 1), and the comparison of the numerical results against field data of a trench excavated in the main channel of the Paraná river near Paraná City, Argentina. Both space and space-time DGFEM methods show a very good agreement between model simulations and analytical solutions, and are able to capture travelling discontinuities without generating spurious oscillations.

REFERENCES

[1] Rhebergen S., Bokhove O., and van der Vegt J.J.W. “Discontinuous Galerkin finite element methods for hyperbolic nonconservative partial differential equations”. *J. Comp. Phys.*, In press, 2008.

[2] Tassi P., Bokhove O., and Vionnet C. “Space discontinuous Galerkin method for shallow water flows —kinetic and HLLC flux, and potential vorticity generation”. *Adv. Water Res.* 30(4), 998–1015, 2007.

[3] Tassi P.A. “Numerical modelling of river processes: flow and river bed deformation”. Ph.D. thesis, http://eprints.eemcs.utwente.nl. University of Twente, 2007.

[4] Tassi P., Rhebergen, S., Vionnet C. and Bokhove O. “A discontinuous Galerkin finite element model for river bed evolution under shallow flows”. *Comp. Methods Appl. Mech. Eng.*, submitted, 2007.

[5] van der Vegt J.J.W. and van der Ven H. “Space-Time Discontinuous Galerkin Finite Element Method with Dynamic Grid Motion for Inviscid Compressible Flows: I. General Formulation”. *J. Comp. Phys.* 182, 546–585, 2002.