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Multiple Signal Classification Algorithm for Inverse Imaging of Two-dimensional Scatterers having Special Characteristics

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ABSTRACT

In this paper, we study the applicability of MUSIC (Multiple Signal Classification) algorithm [1] for imaging two-dimensional scatterers [2]. We first look at the behavior of MUSIC algorithm when used to locate small cylinders of special characteristics using TE incidence. We have taken up TE case because TE illumination on non-magnetic dielectric cylinders presents some interesting properties that have not been investigated earlier. Detailed analysis of the sources induced on cylinders and their linear dependency on each other gives a distinct insight into the use of MUSIC algorithm to locate them. Three such interesting cases are discussed. Further, we look at TM incidence on large cylinders and the use of MUSIC to approximately locate them. It is noticed that use of antenna arrangement that provides full aspect to the cylinder can indeed generate information about presence of the cylinders, which can be used as initial guess for the contour optimization algorithms.

1. For a small dielectric cylinder, in TE case, induced dipole sources are the main contributors to the scattering phenomenon (Fig. 1(a)), and pseudospectrum that uses dipole in *x*-*y* plane as the test source is expected to be able to detect the cylinder. However, here we show that by a proper choice of signal subspace, we can detect the cylinder using pseudospectrum that employs line test source as well, even though the induced magnetic line source is not prominent (Fig. 1(b) and Fig. 1(c)).

2. If the cylinders are isotropic circular cylinders, the electric polarization tensors are rank-filled and the cylinders can be detected using traditional MUSIC, which employs an arbitrarily oriented electric dipole in *x*-*y* plane as the test source. However, there might exist cases, by virtue of the geometry or the material properties of some cylinders, such that their electric polarization tensors may not be rank-filled and that for some principal axis of such a cylinder, the source induced is either zero or very feeble, having negligible contribution in the scattering. In such cases, referred to as degenerate cases, if the test dipole used for MUSIC pseudospectrum is not aligned to the actual induced dipoles in degenerate cylinders, the traditional MUSIC fails to locate those cylinders.

2.1 For example, for an anisotropic circular cylinder, if the relative permittivity along one of the axes in *x*-*y* plane is equal to that of background medium, then there is only one independent dipole induced along the other axis (the optic axis). The range of the multistatic matrix is now spanned by the Green's function corresponding to the only independent dipole that is induced in the cylinder. Since this dipole is a definite linear combination of the components along *x* and *y* directions, we expect that only the correct combination of *x* and *y* directed test dipoles should be able to generate the correct pseudospectrum, and no other combination should be able to detect the cylinder correctly (Fig. 2).

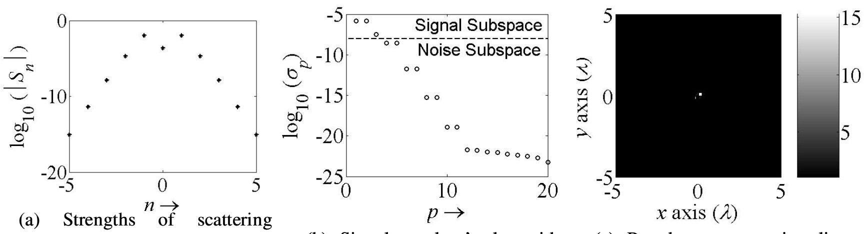
2.2 For an elliptic dielectric cylinder, two independent orthogonal dipoles are induced along major and minor axes respectively, with the strength of dipole along the major axis being stronger than that along the minor axis. As the cylinder becomes sharper and the value of permittivity increases, the dipole induced along major axis becomes more pronounced. In case of a very sharp cylinder and large value of relative permittivity, the dipole along the minor axis is negligible in comparison with the one along major axis, and the cylinder can be said to behave in a degenerate manner in presence of noise (Fig. 3). MUSIC generates results similar to Fig. 2.

Thus, for degenerate cylinders, we need to first determine the direction of induced dipole and then use proper test function corresponding to it in order to detect the cylinder. However, in practice, the presence and exact nature of cylinders cannot be determined using repetitive trial and error methods. Rather, we need a definitive straightforward technique to classify these cases and determine the direction in case of degenerate cylinders. This can be done by using an analytic optimization approach [3].

3. Next, we look at MUSIC as an imaging algorithm for large scatterers, providing a simple, but reliable tool to generate initial guess for optimization algorithms in the form of information about the presence of cylinders and their approximate locations in the domain, even in presence of multiple scatterers with separation of less than one wavelength between them.

3.1 When a small cylinder is illuminated by TM wave, the main induced source is a line current induced at the location of cylinder. However, if the cylinder is large in comparison with wavelength, other multipoles are not negligibly small. Thus, the range of the multistatic response matrix is now spanned by a combination of Green's functions corresponding to different prominent multipoles. The induced sources on a large cylinder are not necessarily distributed evenly [4]. However, roughly speaking, if we place a line source at a point inside the cylinder (as an approximation to the induced multipoles), the radiated field due to it is more likely to be in the range of multistatic response matrix as compared to the field due to a line source placed outside the cylinder. If the pseudospectrum, thus generated using line test source, is conclusive enough to indicate the presence of cylinder, then this very simple approach can be used to approximately locate the cylinder. The result of MUSIC is better if the domain is fully-aspected by the antenna array, rather than a partially aspected domain. However, even in partial-aspect arrangement, a well-aspected domain can provide conclusive results, while a poorly aspected domain may provide inconclusive and inaccurate pseudospectrum.

3.2 The approximate location information generated by MUSIC can be used as an initial guess for optimization algorithms like Differential Evolution Strategy (DES) [5]. The initial guess generated using MUSIC can bring definitiveness to such optimization algorithms and reduce the number of parameters to be optimized, which otherwise waste a lot of computational effort in finding the number of cylinders and locating them in the search space. Fig. 4 shows an example of MUSIC imaging of two large elliptic cylinders with separation less than one wavelength between them and result generated by DES using the initial guess from MUSIC.

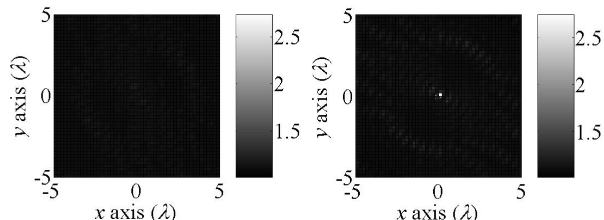


(a) Strengths of scattering coefficients corresponding to the different multipoles induced in a small isotropic dielectric cylinder (highest eleven multipoles are shown here).

(b) Singular values' plot with singular vectors corresponding to first three singular values forming signal subspace.

(c) Pseudospectrum using line test source can detect the cylinder when signal subspace is as in (b).

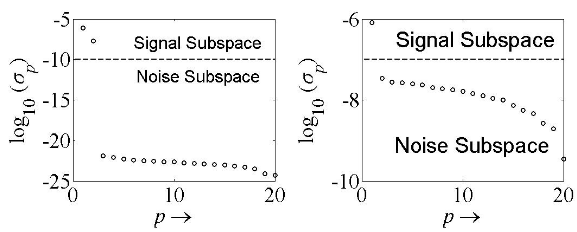
Fig. 1. Implication of choice of signal subspace.



(a) Pseudospectrum using x-directed test dipole cannot detect the cylinder.

(b) Pseudospectrum using test dipole along the optic axis can detect the cylinder.

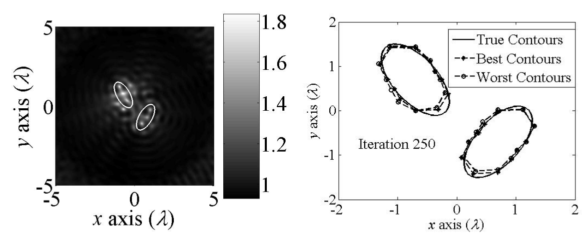
Fig. 2. MUSIC result for anisotropic degenerate cylinder.



(a) Singular values' plot for elliptic cylinder in absence of noise.

(b) Singular values' plot for elliptic cylinder in presence of noise.

Fig. 3. Elliptic cylinder (aspect ratio 100, relative permittivity 40) behaves like a degenerate cylinder in presence of noise (20 dB white Gaussian noise here).



(a) Result of MUSIC for two large elliptic cylinders being investigated under full aspect arrangement and TM illumination

(b) Contour optimization result of 250th iteration of differential evolution strategy.

Fig. 4. Use of MUSIC to image large elliptic cylinders and the result of DES with initial guess as generated using MUSIC.

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