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WHO'S AFRAID OF HIGH INDEX DAES? SCALED AND AUGMENTED LAGRANGIAN FORMULATIONS IN MULTIBODY DYNAMICS

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ABSTRACT

This paper is the latest contribution in a recent line of thought, which identifies all difficulties in the integration of high index DAEs as caused by an unfavorable behavior in the asymptotic behavior of the solution for vanishing time step sizes.

In previously published papers [1,2], it was shown that this undesirable characteristic of the solution can be cured by scaling both the equations of motion and the solution fields. In this paper, we shed some additional light on this important matter, and specifically:

- We show that the scaling can be performed at the continuous level, prior to discretization. This has the effect of curing the problem a priori, irrespectively of the time integration algorithm.
- We introduce scaling factors which depend not only on the time step size but also on the physical properties of the system; this gives an additional beneficial effect to the conditioning of the problem.
- In many multibody formulations, algebraic variables stem from the presence of Lagrange multipliers, but also from the definition of additional algebraic variables such as relative motions. This is a very important aspect in practical applications and was not considered in the previous works on scaling of high index DAEs. We show that in this case it is crucial to consider an augmented Lagrangian formulation together with the scaling.
- We argue that the solution of large scale multibody systems should be performed without pivoting, a requirement for the efficient solution of sparse systems of linear equations, but seldom if ever discussed in the literature. We show that the combination of the proposed scaling combined with an augmented Lagrangian formulation enables the safe use of skyline solvers.

In the full paper, all these facts are rigorously established and demonstrated with the use of numerical examples.

Although further theoretical work is needed before more general conclusions can be drawn, from the results of this and of our previous works on the scaling of DAEs, some clear facts are beginning to emerge:

- High index differential algebraic equations, once put in a properly scaled form, are not more difficult to integrate than ODEs.
- In scaled form, the use of Lagrange multipliers does not cause any additional difficulty. Therefore, unless this leads to computational savings, there is no reason from a numerical point of view to avoid Lagrange multipliers.
- From a numerical point of view, there is not need to use index reduction techniques, which will more often than not lead to methods which enforce the constraints through their higher order derivatives. This will cause possible constraint drifting, a problem which does not affect the direct solution of high index DAEs as considered here.

Although the methods used in most current commercial and research multibody software do not reflect this state of affairs, we believe that the direct solution of high index DAEs has great potential, especially for general purpose finite element based multibody approaches, and will see a more widespread use in the future, as the “fear” of high index DAEs starts to fade in the community.

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