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# ANALYTICAL AND NUMERICAL MODELS FOR BLOCH WAVES AND LOCALISED DEFECT MODES IN PERFORATED PLATES

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## ABSTRACT

We develop a mathematical model dealing with propagation of bending waves in a thin plate perforated by circular holes. It is assumed that the holes form a doubly periodic square array. A spectral problem for the biharmonic operator is formulated in a unit cell, and its analytical solution is constructed by the multipole method. After the numerical treatment of the corresponding algebraic system for the coefficients in the multipole expansion, we construct dispersion diagrams for the two cases where the boundaries of holes are either clamped or free. In particular, for the clamped case there is a total low frequency band gap in the limit of inclusions of zero radius; we also give a simple formula describing the corresponding band diagram in this limit. The formal implication is that the Dirichlet problem for flexural waves in two dimensions cannot be homogenised. On the other hand, we show that holes with free boundaries change the properties of flexural waves only in a perturbative fashion.

We explore the properties of eigensolutions for periodically clamped plates in the zero radius limit. The structure of the dispersion relations is analysed for the first three band surfaces: each of these is constrained to lie between the dispersion surface for the corresponding plane waves. These plane wave surfaces come together in the limit in particular symmetry directions, and the vibrational modes in the pinned plate have the same frequency and group velocity as the corresponding plane waves along these directions.

Finally, dynamic Green's functions are constructed for a finite size crystal created by a square set of pinned points within a thin elastic plate. We create a defect by removing one of the pins and by inducing a time-harmonic displacement of unit amplitude there; the corresponding defect mode is exponentially localised for frequencies within the stop band of the unperturbed structure. The resonance mode, a non-trivial solution of the homogeneous problem, is then determined for the frequency corresponding to the oscillation where the force at the central pin equals zero. We further analyse the transition of the Green's function from the localised to propagating character by displaying Bloch transform patterns for forces acting on the pins.