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RESPONSE SURFACE APPROXIMATION FOR UNCERTAIN STRUCTURAL ANALYSIS

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ABSTRACT

The challenging task in computational engineering is to model and predict numerically the behavior of engineering structures in a realistic manner. Beside sophisticated numerical procedures to map physical phenomena and processes, an adequate description of available data covering the content of provided information is of prime importance. Generally, the availability of information in engineering practice is limited due to available resources. Far beyond the capability to specify crisp values uncertain data are imprecise, diffuse, fluctuating, incomplete, fragmentary and frequently expert specified. Beside objective characteristics like randomness, available data are influenced by subjectivity to a considerable extend. This impedes the specification of unique data models with crisp parameter values to describe the uncertainty. Applying imprecise probabilities objective components of the uncertainty as well as subjective components can be considered simultaneously [1]. A sophisticated procedure to handle imprecise probabilities provide the uncertainty model fuzzy randomness [2]. Since fuzziness, randomness, and fuzzy randomness can be processed simultaneously, it is denoted as generalized uncertainty model.

The consideration of fuzzy random functions $\underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}})$ with $\underline{\mathrm{t}} = \{\underline{\theta}, \tau, \underline{\varphi}\}$ (spatial coordinates $\underline{\theta}$, time τ, further parameters $\underline{\varphi}$) within a structural analysis is referred to as fuzzy stochastic analysis. It is constituted by the mapping of fuzzy random input functions $\underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}}) = \tilde{\mathrm{X}}_1(\underline{\mathrm{t}})$, $\tilde{\mathrm{X}}_2(\underline{\mathrm{t}})$, ..., $\tilde{\mathrm{X}}_l(\underline{\mathrm{t}})$ onto the fuzzy random result functions $\underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}) = \tilde{\mathrm{Z}}_1(\underline{\mathrm{t}})$, $\tilde{\mathrm{Z}}_2(\underline{\mathrm{t}})$, ..., $\tilde{\mathrm{Z}}_m(\underline{\mathrm{t}})$ according to

$$\mathbf{F_{FSA}} : \underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}}) \xrightarrow{\sim} \underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}) \ . \tag{1}$$

The mapping model $\tilde{\mathrm{f}}(\underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}})) = \underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}})$ represents the computational model of the fuzzy stochastic analysis. A numerical realization of fuzzy stochastic analysis is enabled by a bunch parameter representation of fuzzy random functions and is described in detail in [2, 3]. Fuzzy random input functions $\underline{\tilde{\mathrm{X}}}(\underline{\mathrm{t}}) = \underline{\mathrm{X}}(\underline{\tilde{\mathrm{s}}}, \underline{\mathrm{t}})$ as well as fuzzy random result functions $\underline{\tilde{\mathrm{Z}}}(\underline{\mathrm{t}}) = \underline{\mathrm{Z}}(\underline{\tilde{\sigma}}, \underline{\mathrm{t}})$ are expressed in dependency of fuzzy bunch parameter $\underline{\tilde{\mathrm{s}}}$ respectively $\underline{\tilde{\sigma}}$. Introducing $\tilde{\mathrm{m}}(\underline{\tilde{\mathrm{s}}}) = \tilde{\mathrm{f}}_E(\underline{\mathrm{X}}(\underline{\tilde{\mathrm{s}}}, \underline{\mathrm{t}}))$, Eq. (1) is transformed into the mapping

$$\underline{\tilde{\sigma}} = (\tilde{\sigma}_1,\, \tilde{\sigma}_2,\, ...,\, \tilde{\sigma}_{m_1}) = \tilde{\mathrm{m}}(\tilde{\mathrm{s}}_1,\, \tilde{\mathrm{s}}_2,\, ...,\, \tilde{\mathrm{s}}_n) \ . \tag{2}$$

Applying the α-discretization to the fuzzy bunch parameter, an optimization problem is solved in order to determine the α-level sets of the fuzzy bunch parameters ($\tilde{\sigma}_1$, $\tilde{\sigma}_2$, ..., $\tilde{\sigma}_{m_1}$), see also [2, 3]. This algorithm is referred to as fuzzy analysis. Within a fuzzy analysis a stochastic analysis $\mathbf{F_{SA}}$, e.g. applying Monte Carlo simulation, is processed repeatedly. For each realization of the stochastic analysis the deterministic fundamental solution $\mathbf{F_A}$ is carried out.

The repetition of the stochastic analysis and thus of the deterministic fundamental solution increases the computational effort considerable. An improvement of the numerical efficiency can be achieved replacing the expensive analysis by an efficient surrogate model. The idea of such a surrogate model $\mathbf{N}$, also referred to as metamodel, is to approximate the response surface of the respective analysis and to replace the results $\underline{\tilde{Z}}$, if the approximation quality is sufficiently high, by $\underline{\tilde{Z}}^*$. One strategy aims on a substitution of the deterministic fundamental solution $\mathbf{F_A}$ with

$$\mathbf{N_A} : \underline{x}(\underline{t}) \rightarrow \underline{z}^*(\underline{t}) \ . \tag{3}$$

This strategy is straightforward and well-established in the engineering practice [4]. Introducing fuzzy random functions $\underline{\tilde{X}}(\underline{t})$, including fuzzy random fields $\underline{\tilde{X}}(\underline{\theta})$, the dimensionality of a problem may exceed the applicability of metamodels. An alternative strategy substitutes the response surface of the stochastic analysis $\mathbf{F_{SA}}$ by a metamodel.

$$\mathbf{N_{SA}} : \underline{X}(\underline{t}) \rightarrow \underline{Z}^*(\underline{t}) \ . \tag{4}$$

Thereby, metamodels $\mathbf{N}$ based on neural networks are appropriate for the application in engineering. An artificial neural network utilizes the advantages of the human information processing system like complexity, nonlinearity, and parallelism [5]. It is constituted by neurons which are connected by synapses, it has the ability of mapping input signals onto output signals and to adapt to certain tasks during a training phase. The output produced by a neural network may approximate a response surface.

Applying the metamodel $\mathbf{N_{SA}}$ for the response surface of stochastic analysis, the input signals comprise bunch parameters $\underline{s}$ and the network output provides the associated responses in form of bunch parameters $\underline{\sigma}$. Therefore, the network first needs to learn the features of the underlying mapping of Eq. (2). This learning is based on initially performed stochastic analyses $\mathbf{F_{SA}}$. An appropriate constitution of a neural network provide a feedforward neural network and reasonable combination thereof, e.g., committee machines, composite networks, and patchwork networks [4].

The applicability of the introduced procedure is demonstrated by means of example.

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