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LEVEL-SET BASED TOPOLOGICAL OPTIMIZATION FOR STEADY-STATE NAVIER-STOKES FLOW

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ABSTRACT

Since Pironneau's pioneering work [1] in shape optimization for incompressible viscous flow, substantial attention has been attracted to this area for its obvious benefits to fluid-related structures. In this respect, the boundary representative [2] and density-based topology optimization methods [3,4] have symbolized two most intuitive and prevalent methods for fluid design problems. However, the shape optimization lacks of an important function in identifying new topologies and does not guarantee an overall optimum, while the density-based topology optimization commonly leads to non-smooth interfaces and can cause the representation of fluid boundary inaccurate.

Aiming at these two issues, this paper presents a level-set method [5] for topology optimization of steady-state Navier-Stokes flow subject to a specific fluid volume constraint. The solid-fluid interface is implicitly characterized by a zero-level contour of a higher-order scalar level set function and can be naturally transformed to other configurations as its host moves. A variational form of the cost function is constructed based on the adjoint variable and Lagrangian multiplier techniques. To avoid violating the volume constraint, the Lagrangian multiplier derived from the first-order approximation of the cost function is amended by the bisection algorithm. The procedure allows evolving initial design to an optimal shape and/or topology via solving the Hamilton-Jacobi equation.

If a porous media is made up by periodically repeated micro representative volume elements (or base cells), in which shape the base cell can make the porous media maximally permeable. By defining the cost function as the square of horizontal fluid velocity and considering the square-symmetry of the base cell, we get the optimal structure shown as in Fig. 1e, similar to those obtained from the density-based methods in [6]. The initial values for this example is random distributed solid circles whose evolution process to optimal structure is partially depicted by subfiugres in Fig. 1.

The abovementioned level-set method for material design with periodic boundaries can be extended to other more general topology optimization problems in fluid mechanics. For example, if the cost function is dissipation energy due to viscous stress, what an initial channel with a solid circle in its centre will become so that the energy loss is minimal when fluid flows from left to right side through the channel? From the results

shown in Fig. 2, the fluid seems washing away the solid phase from the head of circle and gradually pushes the solid phase backwards until it reaches the right edge. In the mean time, the solid material emerges as a bullet-like shape to minimize flow resistance. The value of the cost function hits the minimum before majority of solid phase reaches the right edge. After that, as the boundary conditions are changed drastically, the cost function goes up rapidly.

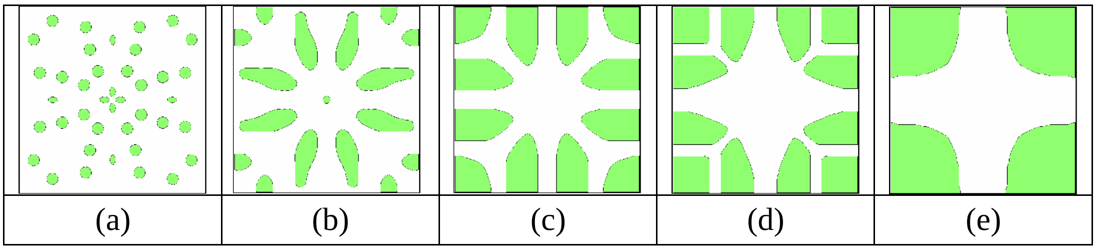

Figure 1. Evolution process for the structure with maximal permeability flow starting from random initial values

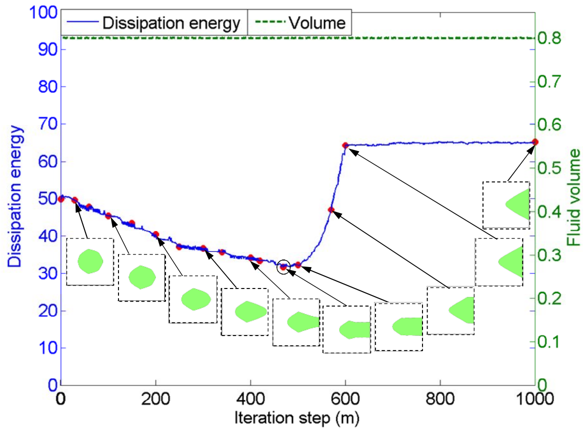

Figure 2. Evolution process for the bullet-like structure with the minimal energy dissipation

REFERENCES

[1] O. Pironnea, "Optimum profiles in Stokes flow," *J. of Fluid Mech.*, Vol. **59**, pp. 117-128, (1973).

[2] M. Y. Wang, X. M. Wang, and D. M. Guo, "A level set method for structural topology optimization," *Comput. Meth. Appl. Mech. Eng.*, Vol. **192**, pp. 227-246, (2003).

[3] G. P. Steven, Q. Li, and Y. M. Xie, "Evolutionary topology and shape design for general physical field problems," *Comput. Mech.*, Vol. **26**, pp. 129-139, (2000).

[4] T. Borrvall and J. Petersson, "Topology optimization of fluids in Stokes flow," *Int. J. Num. Meth. Fluids*, Vol. **41**, pp. 77-107, (2003).

[5] S. Osher and J. A. Sethian, "Front propagating with curvature dependent speed: algorithms based on Hamilton-Jacobi formulations," *J. of Comput. Phys.*, Vol. **78**, pp. 12-49, (1988).

[6] J. K. Guest and J. H. Prévost, "Design of maximum permeability material structures," *Comput. Meth. Appl. Mech. Eng.*, Vol. **196**, pp. 1006-1017, (2007).