8th. World Congress on Computational Mechanics (WCCM8)
5th. European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 – July 5, 2008
Venice, Italy

Very high-order Godunov-type methods on unstructured hexahedral meshes in three space dimensions

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Key Words: *Finite-Volume, Very High-Order, Unstructured Meshes, Hexahedral, WENO*

ABSTRACT

Hyperbolic conservation laws model a wide variety of phenomena in aerodynamics, acoustics, turbulence modelling, environmental sciences etc. The complexity of the equations means that very few analytical solutions are available and numerical methods have to be used. Academic and practical applications in science and engineering often require that these methods be able to accurately represent complex multidimensional geometries, have very high order of accuracy in both time and space (at least third order) and show non-oscillatory behaviour at discontinuities of the solution. Computational efficiency is also a very important issue, especially in three space dimensions.

So far, the development of very high-order finite-volume methods on unstructured meshes has been carried out for the triangular or tetrahedral elements and in two spatial dimensions mostly. The majority of the 3^{rd} and higher-order finite volume for three dimensional problems is implemented on tetrahedral elements. The resulting methods have been successfully applied to a number of problems but their computational efficiency is poor when compared to their structured counterparts.

The goal of the present work is to develop very high-order non-oscillatory finite-volume methods on unstructured hexahedral meshes in multiple space dimensions. The hexahedral elements have twice the node count of tetrahedral one and require fewer elements to discretize spatial domains at the same resolution. The difference in efficiency becomes especially evident for the resolution of boundary layers. The key ingredient of our approach is a polynomial reconstruction scheme based on hierarchical orthogonal basis functions. The reconstruction is made non-oscillatory by using WENO-type procedure. Explicit Runge-Kutta methods are used for temporal discretization.

We show the numerical results of our schemes as applied to the scalar advection equations as well as the compressible Euler equations in three space dimensions. These results illustrate the very high order of accuracy as well as non-oscillatory properties of the schemes.