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HOMOGENEISATION AND DOMAIN DECOMPOSITION WITH APPLICATIONS TO SHELLS MADE OF MICROSTRUCTURED MATERIALS.

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ABSTRACT

Homogeneisation techniques aim to solve problems where there is a significant separation of scales between the global macroscopic problem and the local heterogeneities governing the response of the constitutive materials. They are based on the notion of representative volume elements (RVE), which are microscopic samples of the system under study. Their size H is very small compared to the macroscopic characteristic length L of the global problem, and may be large compared to the size ε of the heterogeneities $L >> H >> \varepsilon$. Each sample is solved at a microscopic scale taking as boundary conditions uniform or periodic displacement data deduced from the averaged values of the solution and its gradient as observed at macroscopic scale. The stress field computed at the microscopic scale is then averaged and used in the solution of the macroscopic problem. Such a strategy remains arbitrary with respect to

- the choice of the ratio between the size of the RVE (representative volume element Ω_H) and the size of the heterogeneities,
- the choice of the boundary conditions to be imposed on the local problem to be solved on each RVE,
- the construction of the local geometry and material coefficients inside the RVE, which are difficult to identify even when using a stochastic representation of the local volume.

Domain decomposition gives a first tool to assess the above strategy. As it will be explained in the talk, a domain decomposition interpretation of an homogeneized solution $\underline{x}_0$ builds a partition of the global domain into

$$\Omega_0 = \cup_k \Omega_{kH},$$

constructs local strain averages associated to the available solution finds the solution $\underline{x}_{kH}, \underline{\underline{\sigma}}_{kH}$ of the local problem on each subdomain Ω_{kH} with imposed averaged strain tensor and defines a discontinuous global solution $(\underline{x}_H, \underline{\underline{\sigma}}_H)$ by juxtaposition of the local fields

$$\underline{x}_H|_{\Omega_{kH}} = \underline{x}_{kH}, \quad \underline{\underline{\sigma}}_H|_{\Omega_{kH}} = \underline{\underline{\sigma}}_{kH}. \tag{1}$$

The problem is then to estimate the distance $(\delta \underline{x}, \delta \underline{\underline{\sigma}})$ between the assembly $(\underline{x}_H, \underline{\underline{\sigma}}_H)$ of local solutions and the exact solution $(\underline{x}, \underline{\underline{\sigma}})$. Since the proposed solution may be discontinuous across interdomain boundaries, we must first propose a mortar based mathematical framework which handles discontinuous fields and then build an adjoint problem whose simplified solution estimates the error level in the homogeneisation strategy.

But new problems appear when dealing with shells made of microstructured materials.

- the shell thickness is at the scale of the representative volume element Ω_H,
- macroscopic bending strains are not directly related to simple gradients of the macroscopic solutions and are not easily translated into boundary conditions to be imposed on the microscopic problem.

Specific actions must be taken both at the macroscopic and at the microscopic scale.

The kinematics at macroscopic scale must first be properly defined, together with the part of the stored energy which can be resolved at macroscopic scale. For a fiber reinforced microstructured material, this means that we must revisit the definition of the bending strains, and verify that they can be properly identified at a macroscopic scale.

The microscopic domains Ω_H must then be constructed, using one domain through the thickness. The remaining issue is then to properly specify the boundary conditions which must be inherited from the macroscopic model. This is highly problem's dependent. Different solutions are currently being tested and will be presented at the conference.

REFERENCES

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