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A 3-D ELASTODYNAMIC SYMMETRIC GALERKIN BOUNDARY ELEMENT FORMULATION FOR SEMI-INFINITE DOMAINS

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ABSTRACT

The understanding and modelling of wave propagation phenomena are of importance in many engineering applications. Numerical solutions of those phenomena are mostly achieved by appropriate discretization schemes. Hereby, the most notably numerical methods are the Finite Element Method (FEM) and the Boundary Element Method (BEM). While both methods are quite sufficiently developed for finite bodies, i.e., bodies which exhibit a closed surface geometry, there is still some need for reliable numerical formulations when semi-infinite domains are under consideration. Characteristically, in such a domain only the outward propagation of waves appear and no inward propagation is possible.

Here, the BEM is the method of choice for dealing with wave propagation in semi-infinite domains. This is due to the fact that, in opposite to the FEM, Sommerfeld's radiation condition [7] is implicitly fulfilled by the underlying time-dependent Boundary Integral Equations (BIEs).

The time discretization of those BIEs can be performed in two different ways. Firstly, if time dependent fundamental solutions are available, the usage of ansatz functions with respect to time yields a time stepping procedure after an analytical time integration within each time step [4]. Secondly, the Convolution Quadrature Method (CQM) developed by Lubich [3] can be used to establish the same time stepping procedure as obtained by a direct time integration [5]. Contrary to the analytical time integration this methodology uses only the fundamental solutions given in Laplace domain which, finally, results in a quadrature formula for the time-domain convolution integrals. This benefit extends the usability of time-dependent Boundary Element Methods also to problems where fundamental solutions are not known explicitly in time-domain [6]. Hence, the CQM will be the preferred time discretization scheme within this work.

Analogous to the time discretization, there exist also several methods for the spatial discretization of the underlying BIEs. Nowadays, the collocation method is still the discretization scheme mostly used within the engineering community. But in recent years the Galerkin type discretizations (for an overview

see [1]) gain more importance since they have been proven to exhibit better stability properties than the collocation method. To obtain the Galerkin Boundary Element Method a variational formulation very similar to Finite Element approaches is applied to the semi-discrete BIEs.

Moreover, it is possible to obtain also a symmetric Galerkin Boundary Element formulation. To achieve this, in addition to the 1st BIE also the 2nd BIE is required. This BIE involves hypersingular kernel functions which must be treated carefully in the numerical implementation. Hence, a regularization based on integration by parts of the elastodynamic fundamental solution is presented which, finally, results in an Boundary Element formulation containing at least only weakly singular kernel functions [2].

In Boundary Element Methods semi-infinite domains are commonly approximated in space just by considering a truncated surface which is supposed to represent a sufficiently large enough region. Unfortunately, applying this procedure to the symmetric formulation implies the evaluation of additional terms on the artificial surface's boundary due to the regularization of the involved kernel functions.

Therefore, a methodology based on infinite elements will be presented to overcome this drawback yielding a symmetric Galerkin formulation which is capable to model wave propagation phenomena also in semi-infinite domains. After a short discussion of some necessary implementation details, both methods, the proposed symmetric Galerkin approach and the more common collocation approach are compared with respect to the spatial and temporal discretization.

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