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A brick element for finite deformations with inhomogeneous mode enhancement

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ABSTRACT

A new enhanced assumend strain finite element for finite deformations is introduced. The element, in the following called Q1/EI9, is based on the split of the deformation of an element into a homogeneous and inhomogeneous part. Enhancement is applied only to the inhomogeneous part of the deformation. The idea of splitting the deformation into a homogeneous and an inhomogeneous part was first introduced by [1] in the context of the Cosserat point element. As many other elements based on the enhanced assumend strain method ([2]), the element introduced here makes use of the Hu-Washizu variational principle in terms of the deformation $\boldsymbol{x}$, the displacement gradient $\boldsymbol{H} = \boldsymbol{F} - \boldsymbol{1}$ and the first Piola-Kirchhoff stress tensor $\boldsymbol{P}$. Here, $\boldsymbol{F}$ is the deformation gradient.

In this approach, an additive split of the displacement gradient $\boldsymbol{H}$ is used,

$$\boldsymbol{H} = \bar{\boldsymbol{H}} + \hat{\tilde{\boldsymbol{H}}}, \quad \bar{\boldsymbol{H}} = \frac{1}{V} \int_{\Omega} \boldsymbol{H} \, \mathrm{d}V \tag{1}$$

with the homogeneous part of the displacement gradient given by the volume average of $\boldsymbol{H}$. The strain energy density function W is split additively into its homogeneous and inhomogeneous part

$$W(\boldsymbol{H}) = W_{\mathrm{hom}}(\bar{\boldsymbol{H}}) + W_{\mathrm{inh}}(\hat{\tilde{\boldsymbol{H}}}) \quad . \tag{2}$$

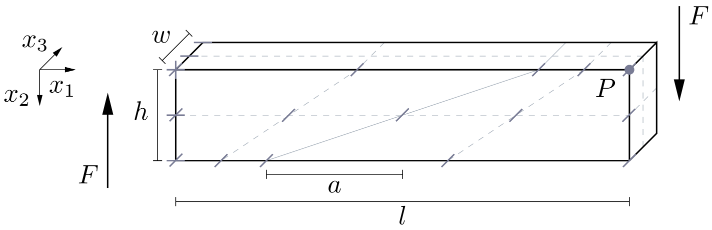
For the homogeneous part of the strain energy density function a compressible Neo-Hooke material is used, while the inhomogeneous part of the strain energy density function is given by a linear relation with a constant elasticity tensor. The inhomogeneous part of the displacement gradient is enhanced such that

$$\hat{\tilde{\boldsymbol{H}}} = \tilde{\boldsymbol{H}} + \hat{\boldsymbol{H}}, \quad \tilde{\boldsymbol{H}} = \boldsymbol{H}(\boldsymbol{x}) - \bar{\boldsymbol{H}} \quad . \tag{3}$$

For the enhancement of the inhomogeneous part of the displacement gradient $\hat{\boldsymbol{H}}$ three quadratic functions fulfilling the orthogonality condition imposed by the Hu-Washizu principle are chosen to interpolate the enhanced modes, as introduced in [2].

The performance of the Q1/EI9 element is studied by comparing it to the standard trilinear (Q1) and triquadratic (Q2) elements as well as the QM1/E12 element [3] and the mixed Q1P0 element for finite deformations [4], which performs well for incompressible materials.

As shown below, the Q1/EI9 element performs very well for shear applied to an irregularly meshed beam, it converges faster than the Q2 element and the Q1 element, which shows severe locking behavior.



Geometry
$l = 10\,\mathrm{mm}$
$h = 2\,\mathrm{mm}$
$w = 1\,\mathrm{mm}$
$a = 3\,\mathrm{mm}$

Load
$F = 6\,\mathrm{N}$

Material
$\Lambda = 600\,\mathrm{MPa}$
$\mu = 600\,\mathrm{MPa}$

Degrees of freedom	Q1/EI9	Q1	Q2	QM1/E12
664	1.0379	0.5778	1.0128	1.0299
4112	1.0314	0.7840	1.0257	1.0279
28576	1.0283	0.9358	1.0270	1.0273
212288	1.0275	1.0007	1.0271	1.0272

Irregularly meshed beam: System, load, material data and deflection of point P

By means of pressure applied to the top of a nearly incompressible block, it is shown that the Q1/EI9 element is not only locking free but also as robust as the Q1P0 element especially suitable for this test, while the QM1/E12 element shows unphysical hourglassing behavior.

Finally, a surface buckling test shows that in contrast to e.g. the QM1/E12 element the Q1/EI9 element succeeds in detecting the correct instability points and modes.

Hence, the Q1/EI9 proves to be a versatile, robust and fast element without the drawbacks of locking and hourglassing.

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