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MODEL ORDER REDUCTION OF LARGE-SCALE SYSTEMS IN FINITE ELEMENT METHOD

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ABSTRACT

This paper presents a novel model oder reduction technique for a mechanical system modelled by linear time-invariant second-order system

$$\begin{aligned} M\ddot{q}(t) + K\dot{q}(t) + Sq(t) &= B_2 f(t), \\ y(t) &= C_1\dot{q}(t) + C_2 q(t), \end{aligned} \tag{1}$$

where $M, K, S \in \mathbb{R}^{n\times n}$ are large and sparse mass, damping and stiffness matrices, $q(t), \dot{q}(t), \ddot{q}(t) \in \mathbb{R}^n$ are displacement, velocity and acceleration vectors, $f(t) \in \mathbb{R}^m$ is vector of excitation forces, $y(t) \in \mathbb{R}^m$ is system output and $C = \begin{pmatrix} C_1 & C_2 \end{pmatrix} \in \mathbb{R}^{m\times n}$, $B_2 \in \mathbb{R}^{n\times m}$ are output and input matrices, respectively. Typically the model (1) is obtained utilizing finite element method and is characterized by thousands or millions of equations and variables. Simulation, optimization and real-time controller design of such large-scale systems is unfeasible within a reasonable computation time, which motivates model order reduction (MOR), i.e., approximation of the original (large) model with the smaller one. After substituting $x^T(t) = \begin{pmatrix} \dot{q}^T(t) & q^T(t) \end{pmatrix}$, the system (1) is rewritten as a first-order generalized state space system

$$E\dot{x}(t) = Ax(t) + Bf(t), \qquad y(t) = Cx(t), \tag{2}$$

which is reduced to $\tilde{E}\dot{\tilde{x}}(t) = \tilde{A}\tilde{x}(t) + \tilde{B}f(t), \quad \tilde{y}(t) = \tilde{C}\tilde{x}(t)$, where $\tilde{E} = W^TEV$, $\tilde{A} = W^TAV$, $\tilde{B} = W^TB$, $\tilde{C} = CV$, and $W, V \in \mathbb{R}^{2n\times r}$ are suitably chosen projection matrices so that $r << n$.

In large-scale setting, this is usually accomplished by the Krylov methods (Lanczos and Arnoldi algorithms), where the projection matrices and/or the reduced system matrices are determined iteratively. In such case, the so-called *moments* (transfer function and possibly some of its derivatives) of the reduced and the original model match at a certain number of points in complex plane, which are refered to as the *interpolation points*. Although numerically very efficient, the Krylov mehods have a drawback that there are no *a priori* guarantees that the reduced model will perserve stability and passivity of the original model. This has further motivated a research in passivity-preserving MOR.

This paper proposes a novel construction of the projection matrices for model ored reduction. As a first step, the system (2) is argumented with an artificial feed-through term so that the second equation in (2) becomes $y(t) = Cx(t) + Df(t)$, where $D \in \mathbb{R}^{m \times m}$. The projection matrices are then derived for the altered system and applied in MOR of the original system. By varying the matrix D we are able to influence the interpolation points, and therefore the cresponding projection matrices. We exploit this aditional freedom to obtain optimal appriximation of the original system. Moreover, the proposed MOR technique is guaranteed to preserve passivity of the original system. In some more detail, the proposed MOR technique is described as follows.

Our choice of optimal set of interpolation points is strongly motivated by Gugercin's result [1] for the H_2 norm of the error system in the Lanczos procedure. This result implies that, in order to obtain an optimal reduced model in H_2 norm, one should choose interpolation points as mirror images $-\lambda^*$ of the original system's poles λ accross the imaginary axis. To render the reduced system stable and passive, the projection matrices are determined as in [2]. The key for preserving passivity, as observed in [3], is choosing *spectral zeros* of transfer function $G(s)$ of the original system as interpolation points. In [2] this is accomplished without explicit computation of spectral zeros, solving generalized partial real Schur decomposition $\mathcal{A}Q = \mathcal{E}QR$ where

$$\mathcal{A} = \begin{pmatrix} A & 0 & B \\ 0 & -A^T & -C^T \\ C & B^T & D + D^T \end{pmatrix}, \qquad \mathcal{E} = \begin{pmatrix} E & 0 & 0 \\ 0 & E & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3}$$

In such case, spectral zeros are equal to the eigenvalues of real quasi-upper triangular matrix R, i.e., the generalized eigenvalues of the pair $(\mathcal{A}, \mathcal{E})$. Columns of Q, where $Q^T Q = I_r$, span an invariant subspace corresponding to the selected set of r spectral zeros. Finally, after some transformations, the projection matrices W, V are obtained from Q. By the appropriate choice of D in (3), the spectral zeros in the right-half of the complex plane approach the generalized eigenvalues of the pair $(-A^T, E)$, which are mirror images of the poles of the original system (2). Taking into account the passivity constraint, the reduced model is thus optimal in H_2 norm.

To evaluate its accuracy and efficiency, the proposed approach has been applied on the finite element model of a simply-supported thin square plate with four inputs and four outputs. After selecting the appropriate feed-through matrix D, generalized partial Schur decomposition (3) has been performed using ARPACK [4], followed by calculation of the projection matrices and the reduced model. Frequency-domain response of such reduced model has been compared to the response of the original system and to the response of another reduced model obtained via balanced truncation method. The obtained results cearly illustrate efficiency of the proposed MOR technique.

REFERENCES

[1] S. Gugercin and A. C. Antoulas. "An H_2 error expression for the Lanczos procedure". *Proceedings of 42nd Conference on Decision and Control*, Vol. **2**, 1869–1872, 2003.

[2] D. C. Sorensen. "Passivity preserving model reduction via interpolation of spectral zeros". *Systems & Control Letters*, Vol. **54**, 347–360, 2005.

[3] A. C. Antoulas. "A new result on passivity preserving model reduction". *Systems & Control Letters*, Vol. **54**, 361–374, 2005.

[4] R. Lehoucq, D. C. Sorensen and C. Yang. *ARPACK users guide: Solution of large scale eigenvalue problems with implicitly restarted Arnoldi methods*, SIAM Publications, Philadelphia, 1998.