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Finite element analysis using time-adaptive Rosenbrock-type methods for finite strain viscoelasticity

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ABSTRACT

In engineering applications there is an increasing necessity to consider polymeric materials, particularly elastomers. Having in mind the high deformability of these materials, one must account finite strains, i.e. geometrical nonlinearities. This contribution is based on a constitutive model of finite strain viscoelasticity. Therein, the deformation gradient is decomposed into a volumetric and isochoric part. The isochoric part of the deformation gradient is multiplicatively decomposed into elastic and inelastic parts. The elastic part defines the stress tensor operating on the intermediate configuration. Moreover, a strain-energy function is proposed dissected into two parts. The first part contains the whole isochoric deformation and the second part exhibiting an internal variable of strain type defined by the multiplicative decomposition of the deformation gradient. Lastly, the additive decomposition of the stresses into equilibrium and overstresses as well as the evaluation of the dissipation inequality leads to constitutive equations for the introduced stresses. According to Hartmann and Neff [4] an elasticity relation, based on a polyconvex strain-energy function, for the description of the equilibrium stress state, is applied. To describe non-linear rate dependence, the viscosity in the Maxwell-element is process dependent.

In this article, the proposed constitutive model is investigated in the context of nonlinear finite element analysis. The consistent application of the space-time discretization in the case of quasi-static structural problems yields after the finite element discretization a system of differential-algebraic-equations (DAE), see [1] and [3]. The algebraic part, a system of nonlinear (algebraic) equations, results from the discretized variation principles, whose solution is computed on the so called “global-level”. The differential part comes from the evolution equations of all internal variables at all Gauss-points, which are carried out by integration on “local-level”. The interpretation of such systems as DAE’s allows the application of efficient integration algorithms. In [3] stiffly accurate diagonally implicit Runge-Kutta (SDIRK) methods are applied to the DAE-system allowing an embedded time-adaptive procedure by retaining the structure of common finite element implementations which are based on a displacement control weak formulation and on backward Euler schemes for local time integrations. These time integration procedures yield a coupled system of non-linear equations for each point in time. The iterative solution procedure of solving a linear system within an exterior loop (the coefficient

matrix is called the tangential stiffness matrix and the unknowns are the increment of the nodal displacements) and the local iteration for evaluating the integration step of the internal variables (stress algorithm) is related to the Multilevel-Newton algorithm proposed in [5] and [6]. These procedures, which embraces the Backward-Euler method as a particular case, are expensive because one has to carry out in each stage of a time step, the computation of a coupled system of nonlinear equations.

An efficient alternative are linear-implicit Runge-Kutta methods of Rosenbrock-type, which can be motivated from DIRK methods (see [2] and [7]). The main idea is to linearize the integration formula by the application of one Newton iteration to each stage with appropriate starting values. The Newton-step leads to a system of linear equations for the unknown stage derivatives in consequence of working the Jacobian matrix directly into the integration formula. These methods have the essential advantage that there are neither "global" nor "local" iterations, which effectively reduce the computational costs. Moreover, time adaptivity of the utilized embedded Runge-Kutta methods is also applicable. The lecture investigates the treatment of the Rosenbrock-type methods in the context of finite element approaches based on large strain deformations. A special focus is put on the expense and achievable accuracy of the time adaptive procedure.

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