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A Multiscale Projection Method for Fracturing Solids

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ABSTRACT

In many brittle materials around the crack front of a macrocrack, microcracks develop which have a significant influence on the propagation behavior of the macrocrack. Those microcracks either cause crack shielding or crack amplification. Since they are orders of magnitude smaller than the whole structure under consideration the effect of the microcracks cannot be captured accurately in a single scale computational analysis. A multiscale approach is required. However, many commonly known multiscale strategies like the FE^2 method or methods based on homogenization in general fail when localization phenomena such as softening damage, cracks or shear bands occur.

In this contribution we present a computationally efficient multiscale finite element method for the simulation of fracture processes in two and three dimensions [1]. The method is based on the decomposition of the displacements into fine scale $\boldsymbol{u}^1$ and coarse scale $\boldsymbol{u}^0$ parts with $\boldsymbol{u}^1 = \boldsymbol{u}^0 + \bar{\boldsymbol{u}}^1$ and $\bar{\boldsymbol{u}}^1$ being the fluctuations of the microstructural displacements due to e.g. microcracks. Microcracks are considered explicitly only on the fine scale level whereas macrocracks are considered on the coarse scale as well as the fine scale level. The extended finite element method (XFEM) [2] in combination with the level set method is used to take into account macrocracks and microcracks accurately. Instead of discretizing $\bar{\boldsymbol{u}}^1$ on the fine scale level, we discretize $\boldsymbol{u}^0$ on the coarse scale and $\boldsymbol{u}^1$ on the fine scale directly using the XFEM approximation

$$\boldsymbol{u}_h^0 = \sum_{I=1}^{n_{\mathrm{n}}^0} N_I^0 \left(\boldsymbol{u}_I^0 + \sum_{j=1}^{n_{\mathrm{enr}}} f_j \boldsymbol{a}_{jI}^0 \right) \qquad \boldsymbol{u}_h^1 = \sum_{I=1}^{n_{\mathrm{n}}^1} N_I^1 \left(\boldsymbol{u}_I^1 + \sum_{j=1}^{n_{\mathrm{enr}}} f_j \boldsymbol{a}_{jI}^1 \right) \tag{1}$$

where $\boldsymbol{u}_I^0$ and $\boldsymbol{u}_I^1$ are the coarse and fine scale standard nodal degrees of freedom and $\boldsymbol{a}_{jI}^0$ and $\boldsymbol{a}_{jI}^1$ are the coarse and fine scale nodal enriched degrees of freedom corresponding to the enrichment functions f_j, respectively. The enrichment functions are chosen in two dimensions according to [2] and in three dimensions according to [3]. The coarse and fine scale test functions $\boldsymbol{\eta}_h^0$ and $\boldsymbol{\eta}_h^1$ are set up in the same

way as the displacements. We assume the material to be isotropic linear elastic. Then, on the fine scale the weak form of equilibrium reads

$$\int\limits_{\Omega^1} \boldsymbol{\sigma}(\boldsymbol{u}^0 + \bar{\boldsymbol{u}}^1) : \mathrm{grad}^{\mathrm{sym}}(\boldsymbol{\eta}^1)\, \mathrm{d}\Omega = \int\limits_{\Omega^1} \boldsymbol{f} \cdot \boldsymbol{\eta}^1 \, \mathrm{d}\Omega \,. \tag{2}$$

Since we prescribe the displacements resulting from the coarse scale computation as boundary conditions on the entire boundary of the fine scale domain, in equation (2) tractions are not considered. For the coarse scale problem we project the stresses obtained from the fine scale calculation onto the respective coarse scale elements. The corresponding weak form for the coarse scale problem then reads

$$\int\limits_{\Omega^0} \boldsymbol{\sigma}(\boldsymbol{u}^0 + \bar{\boldsymbol{u}}^1) : \mathrm{grad}^{\mathrm{sym}}(\boldsymbol{\eta}^0)\, \mathrm{d}\Omega = \int\limits_{\Omega^0} \boldsymbol{f} \cdot \boldsymbol{\eta}^0 \, \mathrm{d}\Omega + \int\limits_{\partial\Omega^0} \boldsymbol{t} \cdot \boldsymbol{\eta}^0 \, \mathrm{d}\partial\Omega \,. \tag{3}$$

The displacement boundary conditions on the fine scale domain boundary are determined by a least square method projecting the displacements from the coarse scale mesh to the nodal degrees of freedom of all boundary nodes of the fine scale domain.

$$\int\limits_{\tilde{\Omega}^1} (\boldsymbol{u}_h^1 - \boldsymbol{u}_h^0) \cdot \boldsymbol{\eta}_h^1 \, \mathrm{d}\Omega = 0 \,. \tag{4}$$

Due to the nature of the enrichment functions, in order to avoid linear dependence, it is necessary to carry out the least square projection (4) over a small strip of fine scale elements along the boundary of the fine scale domain instead of over the surface of the fine scale domain.

In a variety of examples we show that this multiscale projection method works well in two and three dimensions. We show that we can capture the effect of crack shielding and amplification accurately (fig. 1), and we can apply the method to random distributions of microcracks and macrocracks (fig. 2).

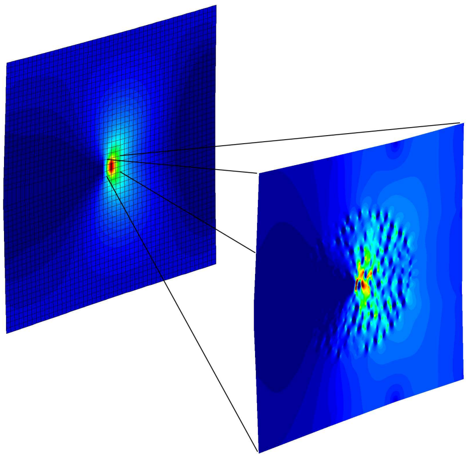

Figure 1: 2D multiscale projection

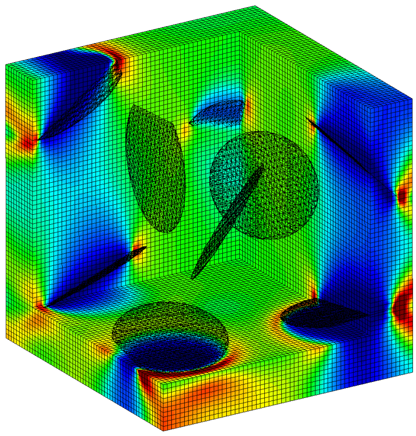

Figure 2: 3D multiple crack problem

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