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SOME CONSIDERATIONS ON THE SECOND ORDER NON INFINITESIMAL, SOLID-RIGID MOVEMENTS ON INCREASED MOBILITY SINGULARITIES OF KINEMATICAL CHAINS

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ABSTRACT

The analysis of singular configurations in mechanisms found in the references is usually focused on the location of these configurations, with the aim of avoiding them as much as possible. But this kind of situations are sometimes unavoidable or even desirable in some applications. This fact leads to the necessity of analyzing exactly how the movement may happen in these configurations. As exposed in [2], in a singular position (here we will name the particular case of increased mobility singularity after the more general term singular position), while the rank of the Jacobian matrix is lower than the one appearing out of the singular configuration, the real mobility of the mechanism cannot be obtained from the nullspace of this Jacobian matrix, because of the existence of infinitesimal motions. These motions can be of second, third or even further order, but, for the general case, the probability diminishes with the order, thus being of most importance the ability to identify the second order case.

As exposed in [1], the second order finite motions can be found as the solution of a quadratic, homogeneous form. Here the problem of the resolution of these equations and the classification of the possible types of solutions that may appear will be addressed, which lead to branching, structural instability and kinematotropy.

For a general configuration in a mechanism, one can write the possible solid rigid movements in the form:

$$\{v\} = a_1\{v_1\} + a_2\{v_2\} + ... + a_n\{v_n\}$$

Where $\{v_1\},\{v_2\},...,\{v_n\}$ are the vectors representing the nullspace of the Jacobian Matrix. The relation among $a_1, a_2, ..., a_2$ for them to represent a non second-order infinitesimal motion is a set of k quadratic homogeneous forms:

$$\begin{Bmatrix} a_1 & a_2 & \dots & a_n \end{Bmatrix} \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} \\ c_{12} & c_{22} & c_{23} & c_{24} \\ c_{13} & c_{23} & c_{33} & c_{34} \\ c_{14} & c_{24} & c_{34} & c_{44} \end{bmatrix}_i \begin{Bmatrix} a_1 \\ a_2 \\ \dots \\ a_n \end{Bmatrix} = 0; i = 1, k$$

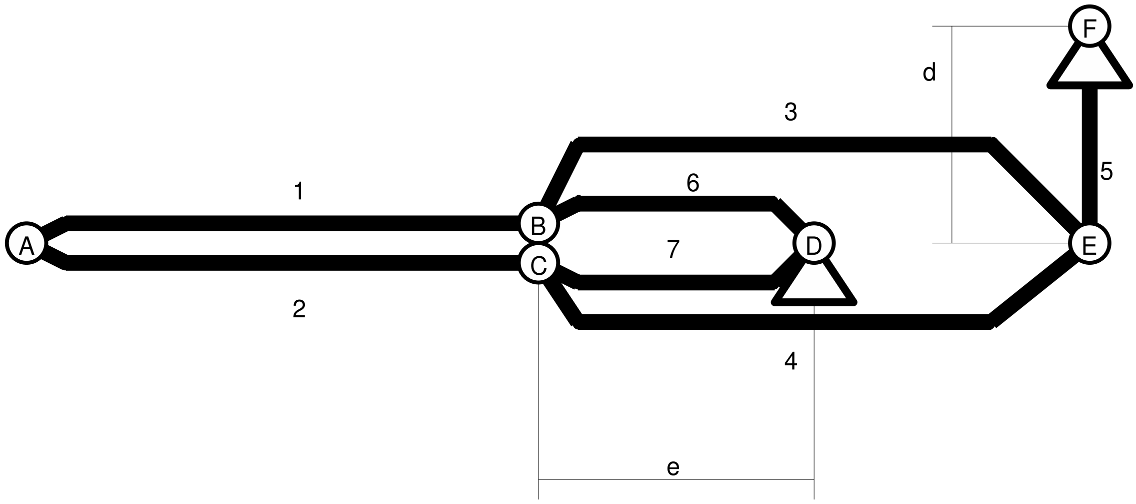
Or, in a more compact expression:

$$\{a\}^T [C]_i \{a\} = 0; i = 1, k$$

By making a change in the system of coordinates, one can write these equations in the form:

$$\{a\}^T [U]_i^T [S]_i [U]_i \{a\} = 0; i = 1, k$$

Where $[S]_i$ is a diagonal matrix and $[U]_i$ represents a change of coordinates. In this new form it is possible to perform a simple analysis of the different possibilities of non infinitesimal motions in the singular configuration. It is easy to demonstrate that the sign of the elements of $[S]_i$ and the amount of zero elements in the main diagonal lead to distinction of unstable structures and branching in the case of $k = 1$, while kinematotropy may only happen in the case of $k > 1$. It is also interesting to note that there is a particular kind of configuration where the singularity happens only in a point, while in most of the cases (for more than one degree of freedom mechanisms) there is a movement that keeps the mechanism in a singular configuration adjacent to the original. Here are some examples that have been correctly identified by the algorithm, representing an unstable structure, and a kinematotropic mechanism.



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