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NONLOCAL DAMAGE MODELLING OF DUCTILE MATERIALS

*** Jana Velde[1], Ursula Kowalsky[1], Tim Zümendorf[1] and Dieter Dinkler[1]**

[1] Institute for Structural Analysis
Technical University of Braunschweig
Beethovenstr. 51, 38106 Braunschweig, Germany
e-mail: J.Velde@tu-bs.de, web-page: http://www.statik.tu-bs.de

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ABSTRACT

For evaluating the time-dependent inelastic stress-deformation behavior of steel structures, a material model is proposed, which is developed within the framework of thermodynamics using a continuum damage approach. Viscoplastic deformation, ductile damage, and localization of the deformation field are considered to describe the material behavior realistically.

The mathematical description of the material behavior is achieved by a set of ordinary differential equations of first order with respect to time. Under assumption of small strains, the elastic strain rate is based on Hooke's law and contains a degradation of Young's modulus by the nonlocal damage variable $\bar{D}$, while the inelastic viscoplastic deformations are obtained following the approach by Chaboche and Rousselier[1,2]. The over-stress σ_{ex}, that determines the onset of viscoplastic deformation, results from a modified criterion based on Gurson[3] and Tvergaard and Needleman[4]. The yield function considers isotropic hardening and volumetric inelastic deformations in case of damage. The evolution of isotropic ductile damage $\dot{D}$ bases on models by Lemaitre et al.[5] and by Gurson[3] and Tvergaard and Needleman[4]. Details concerning the constitutive equations and their thermodynamical consistency are given in Zümendorf[6].

Stress-deformation analysis of structures implies the solution of the underlying initial boundary value problem of the body Ω. The weak form of the equilibrium equation follows the principle of virtual work. Stresses are coupled with strains and internal variables, where the latter characterize the state of the material (hardening, softening, or damage). Softening and damage are accompanied by localization of deformations in small zones. Since the dimension of the zones is not determined, the results suffer from a dependency on the FE-discretization. To overcome this problem, the model is extended by an internal length l_c, which defines the dimension of the localization zone. Here, an implicit gradient enhanced method is applied[8,9], which requires an additional differential equation for the nonlocal formulation of the damage field $\bar{D}$

$$\dot{\bar{D}} - l_c^2 \nabla^2 \dot{\bar{D}} = \dot{D} \tag{1}$$

where

$$\dot{D} = (c_1 + c_2 c_3 e^{-c_3 \epsilon_v^{in}}) \dot{\epsilon}_v^{in} + c_5 (c_4 - \bar{D}) \langle tr \dot{\epsilon}^{in} \rangle \tag{2}$$

with parameters c_1 to c_5 and the equivalent inelastic strain rate $\dot{\epsilon}_v^{in}$. The weak form of the additional balance equation for the nonlocal damage results from weighted residua

$$\int_{\Omega} \left(\delta\bar{D} \left(\dot{\bar{D}} - \dot{D} \right) + l_c^2 \delta\nabla\bar{D}\nabla\dot{\bar{D}} \right) \, d\Omega - l_c^2 \int_{\partial\Omega} \delta\bar{D}\bar{t}_{\bar{D}} \, d\partial\Omega = 0 \tag{3}$$

with Neumann boundary condition $\dot{\bar{t}}_D - n\nabla\dot{\bar{D}} = 0$.

3D-structural analysis is performed applying the finite-element method with trilinear 8 node brick elements for the spatial discretization of the displacement field and the damage field. The accuracy of the material model is evaluated by means of CT-specimens under permanent loading. The model parameters are determined by creep tests on steel specimens (X 20 CrMoV 12.1). The comparison with experiments shows good agreement between the measured and the calculated results. The distribution of damage for CT-specimens with a bore diameter of 10mm and 2mm is shown in Figure 1. In near future the model will be extended to take into account the behavior under dynamic loading.

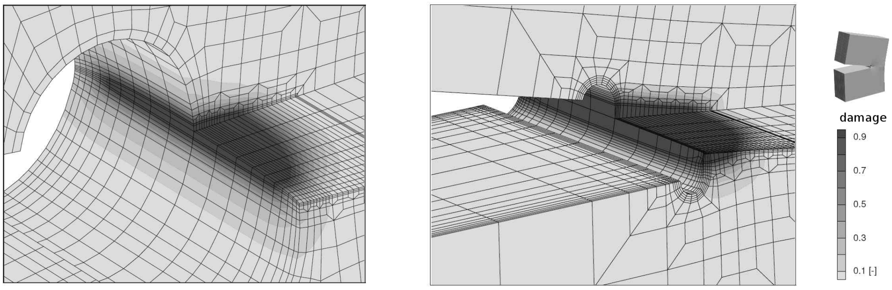

Figure 1: Distribution of damage for CT-specimens with a bore diameter of 10mm and 2mm

REFERENCES

[1] J.L. Chaboche and G. Rousselier. “On the Plastic and Viscoplastic Constitutive Equations - Part I: Rules Developed with Internal Variable Concept”. *J. Pressure Vessel Technology, ASME*, Vol. **105**, 153–158, 1983.

[2] J.L. Chaboche and G. Rousselier. “On the Plastic and Viscoplastic Constitutive Equations - Part II: Application of Internal Variable Concept to the 316 Stainless Steel”. *J. Pressure Vessel Technology, ASME*, Vol. **105**, 159–164, 1983.

[3] A.L. Gurson. “Continuum Theory of Ductile Rupture by Void Nucleation and Growth: Yield Criteria and Flow Rules for Porous Media”. *J. Eng. Mater. Technol.*, Vol. **99**, 2–15, 1977.

[4] V. Tvergaard and A. Needleman. “Analysis of the cup-cone fracture in a round tensile bar”. *Arch. Mech.*, Vol. **32**, 157–169, 1984.

[5] J. Lemaitre, R. Desmorat and M. Sauzay. “Anisotropic damage law of evolution”. *Eur. J. Mech. A / Solids*, Vol. **19**, 187–208, 2000.

[6] T. Zümendorf. *Ein gradientenabhängiges Modell für Schädigung bei viskoplastischem Materialverhalten*, PhD.Thesis, Report-No. **2006-104**, Institute for Structural Analysis, Technical University of Braunschweig, 2006.

[7] R.H.J. Peerlings, R. de Borst, W.A.M. Brekelmans and M.G.D. Geers. “Gradient Enhanced Damage Modelling of Concrete Fracture”. *Mechanics of Cohesive-Frictional Materials*, Vol. **3**, 323–342, 1998.

[8] R.A.B. Engelen, M.G.D. Geers and P.T. Baaijens. “Nonlocal implicit gradient-enhanced elasto-plasticity for the modelling of softening behaviour”. *Int. J. Plast.*, Vol. **19**, 403–433, 2003.