8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A VARIATIONAL DAMAGE MODEL VIA GRADIENT ENHANCEMENT OF THE FREE ENERGY

*** Bojan J. Dimitrijević[1] and Klaus Hackl[2]**

[1] Ruhr-University Bochum
Institute of Mechanics
Universitätstrasse 150
44801 Bochum, Germany
e-mail: Bojan.Dimitrijevic@rub.de
URL: http://www.am.bi.ruhr-uni-bochum.de

[2] Ruhr-University Bochum
Institute of Mechanics
Universitätstrasse 150
44801 Bochum, Germany
e-mail: Klaus.Hackl@rub.de
URL: http://www.am.bi.ruhr-uni-bochum.de

Key Words: *Damage, Gradient Enhancement, Generalized standard media.*

ABSTRACT

We present a thermoelastic damage model that is based on the gradient-type enhancement of the Helmholz free-energy function. Direct introduction of the gradient of the damage variable would require C^1 interpolation of the displacements, which is a complicated task to achieve with quadrilateral elements. Therefore an additional variable field φ is introduced (as in [1]), which serves to transport the values of the damage variable across the element boundaries. Besides the gradient term in the non-local variable φ, the modified free energy function includes a term which penalizes the difference between the non-local and local field:

$$\tilde{\psi} \;=\; \frac{1}{2}\, f(d)\;(\boldsymbol{\epsilon} : \mathbb{C} : \boldsymbol{\epsilon}) \;+\; \psi_\theta \;+\; \frac{c_d}{2}\;\|\nabla\varphi\|^2 \;+\; \frac{\beta_d}{2}\;[\varphi - \gamma_1 d]^2 \tag{1}$$

This makes the model non-local in nature, while preserving C^0 interpolation order of the variables at the same time. The price to pay is an additional set of equations which has to be satisfied on the structural level, involving non-local variables and their derivatives. This strategy results in a pure minimization problem, so that the fulfillment of a LBB condition is not an issue to be considered. We take the interpolation of the displacement field one order higher than the interpolation of the field of additional (non-local) variables. That leads to increased accuracy and removes the post-processing step necessary to obtain consistent results in the case of equal interpolation order (see i.e. [2]).
Standard thermodynamic consideration gives as a result driving forces (stress tensor $\boldsymbol{\sigma}$ and damage conjugate variable q). The stress tensor $\boldsymbol{\sigma}$ maintains the form as in the standard (unenhanced) damage model, while the damage conjugate variable q, on the other hand, contains contributions from two parts:

$$q \;:=\; -\frac{\partial\tilde{\psi}}{\partial d} \;=\; \underbrace{-\frac{1}{2}\, f'(d)\;(\boldsymbol{\epsilon} : \mathbb{C} : \boldsymbol{\epsilon})}_{q_L} \;+\; \underbrace{\beta_d\,\gamma_1\;[\varphi - \gamma_1 d]}_{q_{NL}} \tag{2}$$

The first one (q_L) originates from the common (local) damage considerations. The second one (q_{NL}) reflects the enhancement and is in fact the one that regularizes the model introducing the non-local

influence into the evolution of damage.

The evolution of the damage variable is described following the concept of generalized standard media (as in [3], [4]) through a dissipation potential $D(\dot{d})$ which depends on the rate of the internal variables and retains its common form:

$$D(\dot{d}) \;=\; \sup_q \left[q\dot{d} \;-\; I_K \right] \quad \text{with} \quad I_K(x) = \begin{cases} 0 & \text{if q} \in \text{K} \\ +\infty & \text{if q} \notin \text{K} \end{cases} \tag{3}$$

The set K is defined through a convex inelastic constraint (damage threshold condition):

$$\text{K} = \{q \mid \phi_d(q) \leq 0\} \tag{4}$$

In the present contribution several numerical examples involving different forms of the damage threshold function ϕ_d will be presented in order to illustrate the performance of the presented gradient enhancement strategy. It is shown that the method efficiently removes pathological mesh dependence together with the numerical difficulties connected with the calculation in the softening range of the materials. Moreover, the influence of the additional model parameters (gradient parameter c_d and the penalty parameter β_d) on the global response of the system, distribution of damage and the calculation procedure is discussed. Finally, the extension of the model on other inelastic problems (softening plasticity, combined damage and plasticity), using the same concept, will be presented.

REFERENCES

[1] Dimitrijevic B. J., Hackl K., "A method for gradient enhancement of of continuum damage models", *Technische Mechanik* Vol. **27**, *in Press*, 2007.

[2] Simone, A., Askes, H., Peerlings, R.H.J. and Sluys, L.J., "Interpolation requirements for implicit gradient-enhanced continuum damage models", *Communications in Numerical Methods in Engineering*, Vol. **19**, 563–572, 2003.

[3] Hackl K., "Generalized standard media and variational principles in classical and finite strain", *Journal of the Mechanics and Physics of Solids*, Vol. **45**, 667–688, 1997.

[4] Lorentz, E., Benallal, A., "Gradient constitutive relations: numerical aspects and application to gradient damage", *Computer Methods in Applied Mechanics and Engineering*, Vol. **194**, 5191–5220, 2005.