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A 3D DGFEM solver for the (Reynolds-Averaged) Navier-Stokes equations

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In the framework of the European research project *ADIGMA* project a high-order parallel 3D Discontinuous Galerkin Finite Element Method (DGFEM) solver for the (Reynolds Averaged) Navier-Stokes equations is being developed. The aim is to investigate the applicability of the method to industrial problems, to discern and solve possible bottle-necks in the process, and to tentatively compare the method with state-of-the-art industrial solvers. Bearing this in mind, only the Spalart-Allmaras turbulence model has been implemented, as it is one of the most popular models for external aerodynamics.

Higher-order discretisation methods potentially have huge advantages with respect to current state-of-the-art industrial solvers, which usually emply a finite volume discretisation method (FVM) of relatively low order. With these solvers it is usually very expensive, and often practically impossible to reach grid convergence. Bearing this in mind, it is very likely that the extra cost associated to the quadrature implied in the higher-order methods may be amortized by the faster grid convergence.

This is especially true for the DGFEM discretisation, since the elementwise decoupled interpolation base allows for important economies in the quadrature and iteration techniques. Due to the locality of the interpolation, DGFEM can be considered as a mix between a structured method on the element level and an unstructured method as far as the element interconnectivity is concerned. Moreover the method seems relatively unsensitive to grid quality, code parameters (choice of flux functions, stabilisation parameters etc.) and provides a solid backdrop for (goal-oriented) adaptation strategies. All these qualities combined result in a method which could be particularly well suited for integration in a numerical optimisation chain.

This paper describes the different steps that have been undertaken up to now to cast a DGFEM discretisation into an industrially viable solver.

The spatial discretisation is formulated on generic higher-order curved simplicial elements, and features the interior penalty method for the discretisation of the viscous terms.

We exploit the hybrid structured/unstructured nature of DGFEM by recasting residual operations as matrix-matrix multiplications, for which we use highly optimised BLAS libraries, resulting in a high flop efficiency. In order to further accelerate the quadrature different integration qualities are used in function of the distance to the body of interest, switching from full quadrature on curved elements, over full quadrature on simplices to *quadrature free integration* [1] in the freestream region. The grid consists entirely of curved simplicial elements, which may be reconverted to straight-sided simplices in function of their distance to the region of interest and the validity of the element and its neighbours.

Efficient iterative methods are the backbone of any industrial code. The code mainly uses inexact Newton iterations in combination with a direct solver or GMRES/ILU. In order to exploit the hybrid structured/unstructured nature, an efficient implementation of a block-structured matrix was developed which relies based on BLAS operations to perform the operations on the elementary block level. Research on the extension of the p-multigrid (pMG) strategy [3] to viscous problems is currently in progress. The code is parallellised using a ghost element approach in order to limit and concentrate communications on the one hand and to hide the parallellism from the implementation on the other; the ILU preconditioner is parallellised using using the *Restricted Additive Schwartz (RAS)* approach.

The temporal discretisation uses implicit timestepping methods, in view of the very unfavorable scaling of stable timestep for explicit time-integrators. In order to optimise the number of (sub)time steps [3] and the associated implicit iterations a number of methods is analysed, ranging from the classical three-point backward differencing to *Explicit Step Diagonally Implicit Runge Kutta* (ESDIRK) methods [4].

Applications. The performance of the code is illustrated on a number of stationary/RANS testcases which were identified as critical within the frame of the ADIGMA project, such as the flow around of airfoils at different operating conditions, as well as on two DNS testcases: the vortex street generated by the circular cilinder at Re=100 and the sphere at Re=300.

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