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Accurate and efficient interweaving of substructure uncertainties and component mode synthesis

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ABSTRACT

INTRODUCTION. During the last decade, the interval finite element method (IFEM) has proven to provide the design engineer with useful information on the dynamic behaviour of mechanical structures in the early design stage [1]. Interest in this non-probabilistic approach for uncertainty handling in the product development stage is growing. However, the vertex method, the transformation method, as well as the global optimisation approach, all possible core implementations of the IFEM for dynamic analysis, require a large number of crisp eigenvalue analyses with different values of the uncertain parameters in their respective uncertainty interval. As a result, the IFEM applicability in industrial FE design simulation and validation, with large models and a substantial number of uncertain parameters, is still limited. Therefore, extensive research is dedicated to the reduction of the computational load of the interval FE method.

In the present work, the interval finite element method is combined with the Craig-Bampton component mode synthesis (CMS) method [2]. This numerical reduction technique applies a Ritz-type transformation to each individual component (substructure) of a built-up structure. The deformation of a component is approximated using a limited number of *component modes* (i.e. admissible shape functions), yielding a large reduction in degrees of freedom (dofs) for each component, and thus for the entire structure. In this manner, the computational cost of the FE analysis of large models is drastically reduced. This computational time reduction obtained by the CMS method also benefits the repeated FE analyses required in a non-deterministic context.

A novel approach is presented, which closely interweaves the component mode synthesis technique and the interval finite element method: an approximative component reduction is applied on each uncertain component of an assembly, in each of the FE analyses required in the IFEM procedure. The approach results in an accurate and efficient method for uncertainty propagation in substructuring techniques.

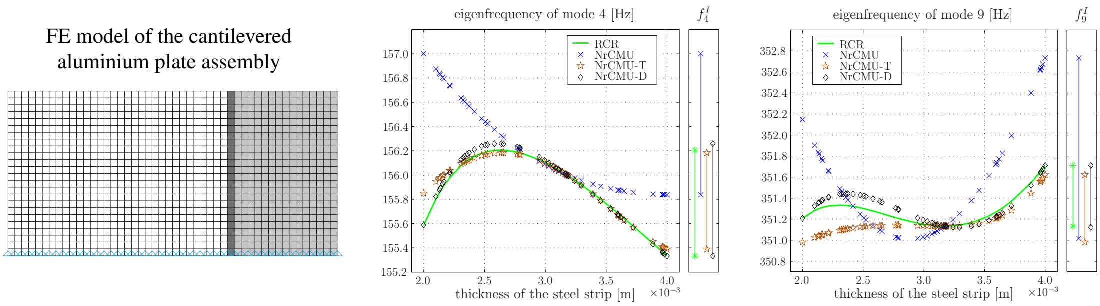
METHODOLOGY. In case of substructured FE models in which uncertainties are present in one or more components to be reduced, the CMS reduction procedure itself is affected by the uncertainties. For each re-analysis of the FE model in an IFEM framework, uncertainties affecting a specific component manifest themselves in, on the one hand, the component mass and stiffness matrices, and on the other hand, in the component modes, which are used in the reduction of the component matrices. In a Repeated Component Reduction (RCR) approach, these entities are recalculated in each FE re-analysis. However,

the calculation of the component modes is a computationally demanding task. Therefore, research has been dedicated to the development of approximative component reduction techniques. Accuracy and efficiency of novel approaches will be compared with the RCR approach.

The *Non-reduced Component Matrices Updating* (NrCMU) approach, developed by the authors, recalculates the mass and stiffness matrices of a component in each re-analysis, but not the component modes. Instead, the component modes of the nominal case are used to determine the reduced component model, resulting in an important computation time reduction. However, in case the uncertain parameters significantly affect the component modes, the dynamic interval results may yield poor approximations. Therefore, two extended NrCMU methods have been developed, which use approximated component modes in the component reduction phase. The first extended method calculates a first-order Taylor series expansion for each of the component modes (NrCMU-T), the second uses deviatoric component modes in a response surface approximation of each component mode (NrCMU-D).

NUMERICAL EXAMPLE. The accuracy of the presented approximative techniques is illustrated on a classical benchmark example. The model consists of a cantilevered aluminium plate, divided into two unequal substructures. The leftmost row of elements of the second substructure is changed to represent a small steel strip (dark grey in the leftmost figure). The finite element model with 7350 dofs was reduced to an equivalent and accurate 168-dof Craig-Bampton model, assuring errors on the lowest 10 eigenfrequencies to be below 0.25% compared to those of the full FE model.

The figure presents results of a non-deterministic eigenfrequency analysis of the plate assembly considering a large uncertainty interval on the thickness of the small steel strip between 2.0 and 4.0 mm. The left-hand side of the results figures shows the change of the eigenfrequency with respect to the uncertain parameter, while the right-hand side depicts the obtained eigenfrequency ranges.



Most eigenfrequencies of the plate assembly display a monotonic increasing behaviour with respect to the uncertain parameter. In general, good approximations for these eigenfrequencies are obtained by the basic as well as the extended NrCMU approaches. Non-monotonic behaviour is found for modes 4 and 9. For these cases, the basic NrCMU method is clearly insufficient: the non-monotonicity of the fourth eigenfrequency is not captured, while the variation of the ninth is highly overestimated. The use of first-order Taylor series expansions of the component modes gives rise to a profound improvement of the eigenfrequency approximations. Best results are obtained with the use of response surface approximations of the uncertain component modes using deviatoric component modes.

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