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A Kriging based optimisation algorithm for interval and fuzzy FRF analysis

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ABSTRACT

Introduction The finite element method is a useful tool to predict the behaviour of a structure under static and dynamic loads. Reliable finite element analyses can reduce the need for prototype testing and thus reduce the design validation cost and time. In many real life situations however, a deterministic analysis is not sufficient to assess the quality of a design. In a design stage, some physical properties of the model may not be determined yet. But even in a design ready for production, design tolerances and production inaccuracies introduce variability and uncertainty. In these cases, a non-deterministic analysis procedure is required, either using a probabilistic or a possibilistic approach.

The authors developed a hybrid (global optimisation and interval arithmetic) interval finite element procedure to predict the bounds on frequency response functions (FRFs) of problems with interval or fuzzy uncertainties [1] and a Kriging based optimisation algorithm, which will be presented at the conference. This paper shows that this combination results is a highly efficient procedure, capable of handling industrial-size models.

Procedure The hybrid procedure, developed by the authors, consists of three major steps. In the *first step*, the ranges of the modal parameters (the inverse of the modal stiffness $\hat{k}^{-1}$, the inverse of the modal mass $\hat{m}^{-1}$ and the eigenfrequency f) are determined for each mode and – for fuzzy analyses – for each α-level, using a global optimisation procedure. This is the computationally most expensive step of the algorithm. In the *second step*, the ranges of the modal parameters determined in the previous step are translated to the modal envelope FRFs for each mode, using an interval arithmetic procedure. In the *third step*, the total envelope FRF is determined by calculating the sum of all modal envelope FRFs calculated in the previous step.

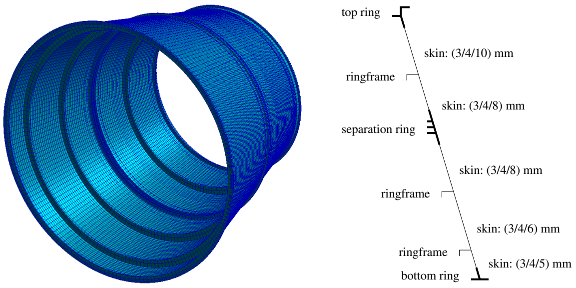
Because the evaluation of the modal parameters requires an FE analysis, which can be computationally expensive itself, and because of the high number of optimisations required, the efficiency of the first step determines the computational cost of the analysis. To analyse anything but very simple models, a highly efficient optimisation algorithm is imperative. Response surface based optimisation techniques

can take advantage of two special properties of the optimisation problem. First, the FE analysis calculates all modal parameters for all modes of interest, so by using the same response points to approximate all objective functions, the computational cost can be decreased up to a factor 100 (depending on the number of modes of interest). Second, the search region at higher α-levels is a subset of the search region at lower α-levels, so only a response surface at the lowest α-level of interest is required. Kriging or DACE (Design and Analysis of Computer Experiments) based techniques [2] are well suited to approximate these smooth, non-linear objective functions. The authors developed a simple and effective but computationally inexpensive algorithm to select the response points, which will be presented at the conference.

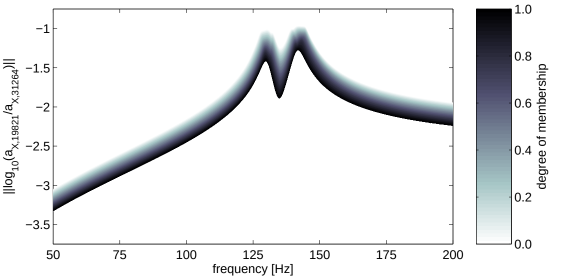
Application The procedure is applied on a stiffened conical shell. The figures show the finite element model (left) and a schematic view (middle). The structure consists of five conical shell rings, connected and stiffened by stiffening rings. The finite element model consists of about 38000 nodes (228000 DOFs) and about 28000 elements. The model is subject to 5 uncertainties: the thicknesses of the shell structures between the stiffening rings, as shown in the schematic view.

This conical structure serves as a connection between two relatively rigid structures, one rigidly bolted to the bottom ring and the other rigidly bolted to the top ring. Since at this design stage, the exact properties of these structures are unknown, they are modelled as rigid body elements. One is connected to all nodes on the bottom ring, the other one is connected to all nodes on the top ring. The acceleration transmittance FRF between the centres of these rigid body elements in the longitudinal direction is calculated.

The preliminary analysis shows that only three modes influence this FRF. Calculated at 6 α-levels $(0.0, 0, 2, \ldots, 1.0)$, this requires 90 optimisations. The optimisation procedure is started with a latin hypercube design of 15 points, to which 3 sets of 5 response points out of a set of 50 candidates are added. In total, only 30 response points are used to perform the 90 optimisations. The figure shows the fuzzy upper bound on this FRF (right). Comparison with a full factorial design of experiments (whose results are approximate too) suggests that the maximum error is around 3%.



FE model (left) and schematic description (right)
of the stiffened conical shell structure



Fuzzy upper bound on the FRF between
the top and bottom ring of the structure

REFERENCES

[1] David Moens and Dirk Vandepitte. Interval sensitivity theory and its application to frequency response envelope analysis of uncertain structures. *Comput. Methods Appl. Mech. Engrg.*, 196:2486–2496, 2007.

[2] Jerome Sacks, William J. Welch, Toby J. Mitchell, and Henry P. Wynn. Design and analysis of computer experiments. *Statistical Science*, 4(4):409–435, 1989.