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Analytical integrations in meshless implementations of local integral equations

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ABSTRACT

The question about using either mesh or not mesh has been discussed recently in the paper [1], but it seems that this question can not be answered definitely because of advantages and disadvantages on both the sides. Even if the standard discretization techniques are applicable to the numerical solution of complex boundary value problems with including more or less modifications, it is still reasonable to develop new formulations by proper modelling of essential physical properties and advanced approximations of unknown physical fields in a consistent way. Simultaneously, the modelling as well as approximations should be as simple as possible without violating previous assumptions.

The strong formulations of the problems of mathematical physics and engineering sciences are given by partial differential equations (PDE). Note that the governing PDE are derived from the integral form of the balance equations valid on an arbitrary small sub-domain. Hence, the consideration of the local integral equations (LIEs) on finite size local sub-domains brings a proper physical interaction among the discrete degrees of freedom represented by the nodal values used in approximations of field variables. The LIE formulations belong to weak formulations. In this paper, we shall consider meshless approximations given by the Point Interpolation Method (PIM) and two variants of the MLS approximation, namely the standard and the Central Approximation Node (CAN) concepts [2]. The LIE formulation is applied on circular shape local sub-domains selected around each nodal point. Since the meshless shape functions are not known in a closed form, the numerical integration approach requires application of certain evaluation procedure at each integration point and the approach is becoming very time consuming. In order to reduce the time (t_m) needed for creation of the system matrix in the discretized formulation, we have tried to perform the required integrations analytically. This can be done by using the second order Taylor series expansion of the shape functions around the centre of the circular sub-domain. Thus, for each approximation approach, we receive two computational techniques based either on the numerical or analytical integration. In the case of continuously non-homogeneous media, the spatial variations of material coefficients are expanded into Taylor series too. It is interesting to compare the computational efficiency of the derived techniques. In

this paper, the comparative study is carried out on simple test problem with existing the exact solution. Let us consider a 2D square domain with uniaxial exponential gradation of the Young modulus and subjected to uniform tension in that direction. Besides the time t_m, we have investigated also the time t_{sol} required for solution of the system of discretized equations, and the total time t_f measured after the solution of the system matrix (i.e. $t_f = t_m + t_{sol}$). Having used uniform distributions of nodes, we received the dependence of t_m and t_f on the h-parameter (the distance of any two nodes) as shown in Fig. 1 and Fig. 2, respectively. As regards the time t_{sol} it is independent of the approach used for creation of the discretized equations.

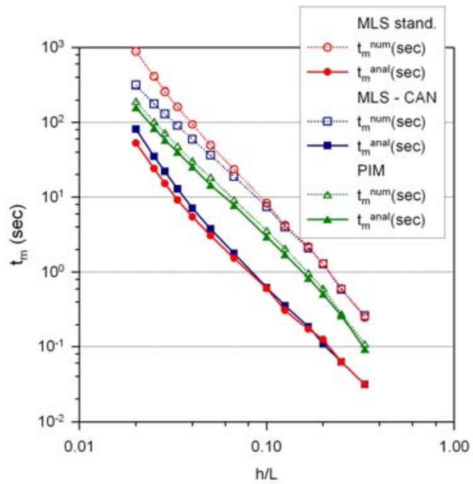

Fig. 1

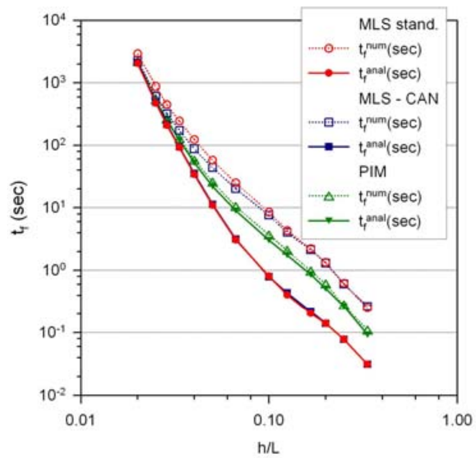

Fig. 2

It can be seen from Fig. 1 that the difference between the times t_m^{num} and t_m^{anal} are more expressive in the case of MLS approximation than in the PIM approach. The explanation of the computational results will be discussed in the paper. Fig. 2 shows that the differences in t_f by standard MLS approach and the MLS-CAN are negligible either in numerical or in analytical integration variant. Moreover, the differences among various techniques diminish with increasing the density of nodes. This can be explained by the fact that t_{sol} is becoming dominant part of t_f with increasing the density of nodes for the employed solver for systems of linear algebraic equations (i.e. $t_{sol} = t_f - t_m \to t_f$, since $t_m / t_f \to 0$ with $h \to 0$).

To get rid of dominant contribution of t_{sol} to t_f, the employed solver should be replaced by a substantially faster one. Then, one could fully realize the benefit of shortening the time (t_m) for creation of the system matrix by analytical integration.

Another conclusion is the conversion of the weak formulation into strong one.

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