8th.World Congress on Computational Mechanics(WCCM8)
5th.European Congress on Computational Methods in Applied Science and Engineering(ECCOMAS 2008)
June 30-July 5,2008
Venice,Italy

TWO-ORDER AND TWO-SCALE ANALYSIS METHOD FOR THE STRUCTURES OF COMPOSITES WITH QUASI-PERIODICITY

FANG SU[†] , **JUNZHI CUI**[†]

[†]Chinese Academy Sciences No.55, East Road Zhongguancun, Beijing, 100080, China
sufang@lsec.cc.ac.cn, cjz@lsec.cc.ac.cn

Key words: Homogenization, Two-order and two-scale, Quasi-periodic elastic structure, Composite materials

Abstract

The homogenization method for elasticity problems was proposed theoretically by Lions, Babuska and other authors. The main idea is to obtain an average field equation by constructing properly local smoothing operator. As a result, one can solve numerically the homogenized equation in a coarse mesh. The multi-scale method, which can capture the local micro-characteristics, was developed for the structures of composite materials with periodicity later. Furthermore, it only needs to compute on one periodic cell, and then save a great computation amount.

In some cases, coefficients reflecting the character of composite materials have not whole-periodicity, but quasi-periodicity. In this situation, the homogenization theory for the quasi-periodic structures of composite materials has been discussed by Lions, and it can not obtain high-order and multi-scale approximation on the solution to use the multi-scale method here before. Cao et gave the first-order approximation and several basic estimations for the problem on quasi-periodic structures. They suggested that the solution of original problem and the numerical solution were equivalent only in integral sense. For the systems of linear elasticity of the structure of composite materials with quasi-periodic micro-structures the two-order and two-scale approximation to the true solution will be presented by introducing a correct term in this paper. The two-order and two-scale approximate solution is equivalent to the true solution of original problem, not only in the homogenization sense, but also in the nearly pointwise sense with $\epsilon^{\frac{1}{2}}$ order. We will demonstrate the validity of the newly approximate solution by mathematical convergence proof and numerical computation.

We consider the mixed boundary value problems for the systems of linear elasticity of the structure of composite materials with quasi-periodic micro-structures as follows:

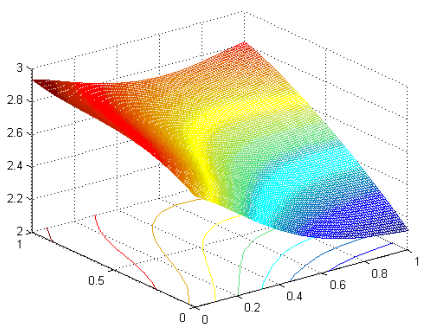
$$\begin{cases} \mathcal{L}_i^\epsilon(u^\epsilon) = \frac{\partial}{\partial x_j}[\omega(x)a^\epsilon_{ijhk}(x)\frac{1}{2}(\frac{\partial u^\epsilon_h(x)}{\partial x_k}+\frac{\partial u^\epsilon_k(x)}{\partial x_h})] = f_i(x) & \text{in} \quad \Omega \\ \mathbf{u}^\epsilon(x) = \bar{\mathbf{u}}(x) & \text{on} \quad \Gamma_1 \\ \sigma^\epsilon_i(x) = \nu_j\omega(x)a^\epsilon_{ijhk}(x)\frac{1}{2}(\frac{\partial u^\epsilon_h(x)}{\partial x_k}+\frac{\partial u^\epsilon_k(x)}{\partial x_h}) = p_i(x) & \text{on} \quad \Gamma_2 \end{cases}$$

where Ω is a bounded domain with Lipschitz continuous boundary; $\omega(x)$ is a function only related to macro-variable x; $a^\epsilon_{ijhk}(x)$(i,j,h,k=1,2,3) are ϵ periodicity, Let $\xi = x/\epsilon$, then $a^\epsilon_{ijhk}(x) =$

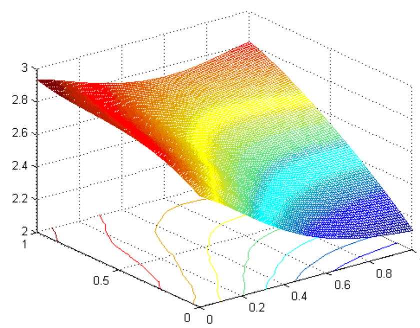
$a_{ijhk}(\frac{x}{\epsilon}) = a_{ijhk}(\xi)$; $\mathbf{u}^\epsilon(x)$is the displacement solution. The two-order and two-scale approximate solution for $\mathbf{u}^\epsilon(x)$ is expressed as follows:

$$\tilde{\mathbf{u}}(x) = \mathbf{u}^0(x) + \epsilon \mathbf{N}_{\alpha_1}(\xi)\frac{\partial \mathbf{u}^0(x)}{\partial x_{\alpha_1}} + \epsilon^2 \mathbf{N}_{\alpha_1\alpha_2}(\xi)\frac{\partial^2 \mathbf{u}^0(x)}{\partial x_{\alpha_1}\partial x_{\alpha_2}} + +\epsilon^2 \mathbf{C}_{\alpha_1}(x,\xi)\frac{\partial \mathbf{u}^0}{\partial x_{\alpha_1}}$$

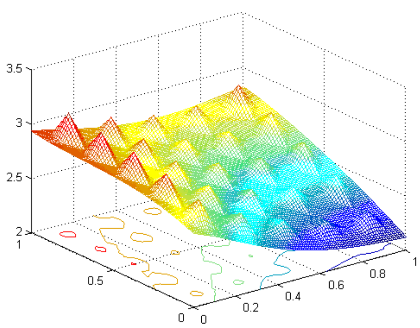
In full paper each term has been determined. An example is demonstrated here to show the validity of the approximate solution.



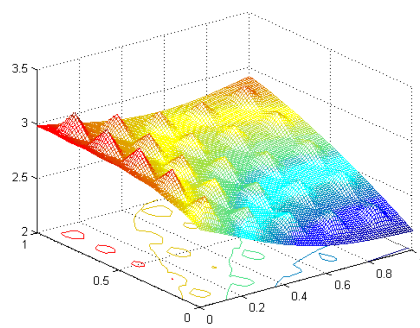
a



b



c



d

Fig.a shows the homogenization solution, Fig.b the one-order approximation solution, Fig.c the two-order correction, Fig.d the finite element solution in very refined meshes. From Fig.a-d, one can see that the newly two-order and two-scale approximation has good consistency with the FE solution in very refined meshes. But the homogenization solution and first-order approximation solution have less approximation.

The damage and fatigue analysis on the structure of composite materials with quasi-periodic micro-structures can lead to above systems of linear elasticity. The damage effect analysis in heterogeneous media has been studied by homogenization method or one-order two-scale approximation, but the works are few. It is well known that the homogenization stress field lose local effects, particularly, local stress concentrations. The two-order and two-scale approximation can capture the local stress concentration, and is better to reflect the true situation for local revolutionary behaviors.

Acknowledgements

The project supported by the Special Funds for Major State Basic Research Pro-ject (2005C B321704) and the National Natural Science Foundation of China (10590353 and 90405016).