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FAST MULTI-LEVEL MESHLESS METHODS COMBINED WITH BOUNDARY INTERPOLATION

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ABSTRACT

The main advantage of the meshless methods of partial differential equations is that they require neither domain nor boundary discretization such as a (Cartesian or curvilinear) grid or finite element mesh. It is sufficient to use a set of boundary points only (and, sometimes, a set of inner points) which, however, do not need to have any structure. The price of this approach that in the vast majority of such methods lead to discrete linear systems with dense, highly ill-conditioned matrices, which can cause serious computational difficulties. For instance, the popular method of particular solutions splits the original problem into two subproblems: to find a particular solution taking into account no boundary conditions and to solve a homogeneous problem with modified boundary conditions. The first task can be treated by e.g. radial basis function (RBF) approach which is based on the inner interpolation points, while the second task can be solved by using a boundary technique e. g. the boundary knot method [1] or the method of fundamental solutions [2]. However, the above subproblems exhibit the same computational disadvantages as the original problem. Moreover, in case of problems which are nonlinear and/or with variable coefficients, both the particular solution and the fundamental solution can hardly be obtained.

The use of the direct multi-elliptic interpolation [3] is a possible way to avoid the above mentioned computational difficulties. Here the interpolation function is defined as a solution of a higher order auxiliary partial differential equation (e.g. the biharmonic equation, the modified bi-Helmholtz equation etc.) supplied with interpolation conditions as special (pointwise) boundary conditions. As shown in [3], the interpolation does not essentially differ from an RBF-like interpolation based on the fundamental solution of the applied higher order partial differential operator. However, from computational point of view, it is much cheaper when using quadtrees and multi-level methods. Moreover, the use of dense and ill-conditioned matrices is avoided. The homogeneous subproblem can be solved in a similar way, using a regularized version of the method of fundamental solutions based on a Laplace-Helmholtz interpolation with the fourth-order operator $\Delta(\Delta - c^2 I)$ with a carefully chosen scaling parameter c as proposed in [4] (here I denotes the identity operator). However, the optimal value of the scaling parameter depends on the local density of the interpolation points, which can vary from location to location. Moreover, the improper definition of c causes numerical singularities on the boundary (if c is too high) or poor approximation of the homogeneous partial differential equation (if c is too small). Both cases result in an increase of the errors.

In our talk, we simplify the approach introduced in [5]. The particular solution is obtained by a direct, quadtree-based multi-level solution of the inhomogeneous problem. The construction of the homogeneous solution is based on a biharmonic interpolation which is performed in a narrow vicinity of the boundary only. This neighborhood is automatically determined by the (unstructured) boundary points using the same quadtree cell generation as in the previous step. In fact, the approach does not require to split the solution of the original problem into finding a particular solution and solving a homogeneous subproblem. The proposed method contains no scaling parameter to be optimized and generates no boundary singularities. The method makes it possible to avoid the use of ill-conditioned matrices. Error estimations will be derived and numerical examples are also presented.

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