8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

New displacement-based superelement for poroelastic materials. Derivation and convergence study.

C. Batifol[1,2], * M. N. Ichchou[1] and M.-A. Galland[2]

[1] Laboratoire de Tribologie et de Dynamique des Systèmes - UMR CNRS 5513
[2] Laboratoire de Mécanique des Fluides et d'Acoustique - UMR CNRS 5509
Ecole Centrale de Lyon, 36 Avenue Guy de Collongue, 69134 Ecully cedex, France
Cedric.batifol@ec-lyon.fr, http://ltds.ec-lyon.fr, http://www.lmfa.ec-lyon.fr

Key Words: *Poroelastic Materials, Component Mode Synthesis, Fluid/Structure Interaction.*

ABSTRACT

Poroelastic materials are widely used in vibroacoustic applications such as sound absorption and transmission. Designing appropriate sound packages involves numerous numerical studies at early stages. Poroelastic materials are then modelled using the biphasic theory developed by Biot [1]. The approximation of the 3D-harmonic poroelasticity problem, using the finite element method (FEM), yields to complex and frequency-dependent matrices. Moreover, the number of degrees of freedom (dofs) per node is at least 4 using the $(\mathbf{u}^s, p)$ formulation [2], whereas the use of $(\mathbf{u}^s, \mathbf{U}^f)$ as kinematic unknowns leads to 6 dofs per node. These drawbacks still prevent intensive optimisation procedures from being performed.

The superelement derived in this contribution expresses the admissible search space for fluid and solid displacements as the direct summation of so-called hybrid dynamic modes and static fluid functions. The frequency-dependent eigenproblem is solved, without simplification, using the Non-Linear Arnoldi (NLA) algorithm [3]. At junction interfaces, hybrid boundary conditions are applied. The solid phase remains free while the fluid phase is fixed. This non-physical linear constraints are applied as Dirichlet conditions using the $(\mathbf{u}^s, \mathbf{U}^f)$ formulation. Resulting eigenmodes are orthogonalised to form the dynamic basis. Finally, static fluid functions are calculated thanks to classical Guyan condensation.

The convergence properties of this superelement are compared to those of more classical Craig & Bampton [4] and MacNeal superelements [5]. A 1D test case is considered. A 12-centimetre length felt sample is subjected to an harmonic imposed pressure or prescribed displacement. The residual error between direct (considered as reference) and reduced calculations is estimated in a least mean square sense. At a forcing frequency of 2 000 Hertz, using the hybrid superelement, only 11 dynamic modes are needed to have an error less than 1 percent for both phases displacements. The figure (1) shows, for all three kinds of superelement, the remaining errors between direct and reduced calculations when one of the eleven dynamic modes is not retained in the projection matrix. The fluid and solid phases convergence rates are the same when using the hybrid superelement (*H-Basis*), which is not the case when using Craig & Bampton (*CB-Basis*) or MacNeal (*MN-Basis*) one. For these two last projection basis, opposite evolution of fluid and solid errors is observed around the fifth mode. The use of hybrid

boundary conditions at the junction interface allows to have relevant information for both fluid and solid displacements in each calculated eigenmode. The dynamic basis of the superelement can then be derived using a simple criterion for mode selection, such as cut-off high frequency. Finally, a 3D test case is considered. The reduction procedure is proved to be efficient and more straightforward to use than classical component mode synthesis techniques.

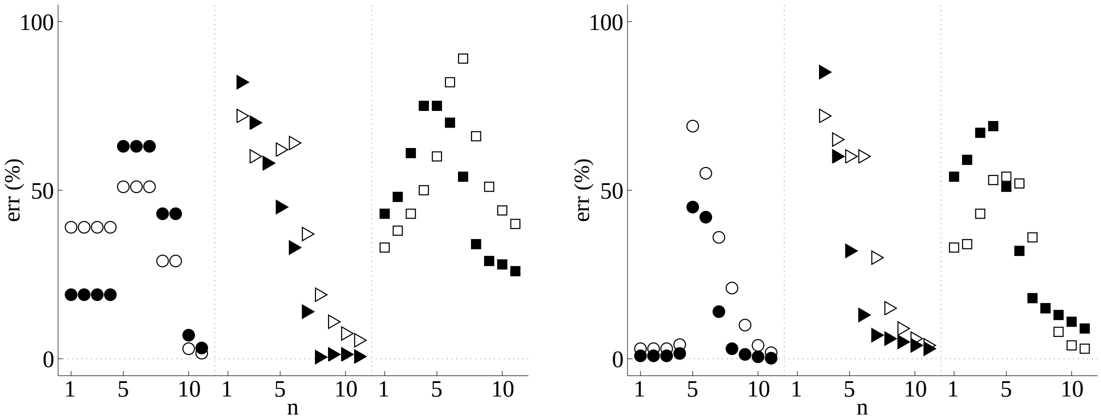

Figure 1

Error evolution versus non-retained modes at frequency 2 000 Hertz for imposed displacement (left) and pressure (right). ○ and ● stand for *H-Basis* solid and fluid errors. ▷ and ► stand for *CB-Basis* solid and fluid errors. □ and ■ stand for *MN-Basis* solid and fluid errors.

REFERENCES

[1] M. A. Biot. ”Theory of propagation of elastic waves in a fluid-saturated porous solid”. *J. acoust. soc. am.*, Vol. **28**, 168–178, 1956.

[2] N. Atalla, R. Panneton and P. Debergue. ”A mixed displacement-pressure formulation for poroelastic materials”. *J. acoust. soc. am.*, Vol. **104**, 1444–1452, 1998.

[3] H. Voss. ”An Arnoldi method for non-linear eigenvalue problems”. *Tech. report, Dept math., TU Hamburd-Harburg*, 2003.

[4] R. R. Craig and M. C. C. Bampton. ”Coupling of substructures for dynamic analyses”. *AIAA Journal*, Vol. **6**, 1313-1319, 1968.

[5] R. H. MacNeal. ”A hybrid method of component mode synthesis”. *J. comput. struct.*, Vol. **1**, 581-601, 1971.