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A posteriori error estimates for fully discrete approximations of time–dependent problems

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ABSTRACT

A posteriori error estimates are succesful mathematical tools to provide efficient adaptive versions of many numerical methods. A posteriori error estimators provide global and local information on the error of the numerical solution, using only the approximate solution and the data of the problem.

In this work we derive a posteriori error estimates of optimal order for fully discrete approximations of linear parabolic problem and for mixed methods for the time–dependent Stokes problem. For the time discretization we consider popular schemes for flow computations as the Crank–Nicolson scheme and the fractional step scheme. Available results of a posteriori error analysis for these schemes are limited.

The main tool of our analysis is an appropriate space-time reconstruction $\hat{\omega}$, of the piecewise linear in time approximate solution U. We derive a posteriori error estimates of optimal order for the norms $L^\infty(0,T;L^2(\Omega))$ and $L^\infty(0,T;H^1(\Omega))$ by the comparing of the exact solution u to the reconstruction $\hat{\omega}$.

In this work the control of spatial errors is based on using the *elliptic reconstruction* approach [4], and time estimators are based on an appropriate extension of ideas in [1]. Related works include [2], [3].

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