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YIELD SURFACE FOR ELASTOPLASTIC BEAM 2D ELEMENT CONSIDERING DAMAGE MATERIAL

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ABSTRACT

A procedure for determining the yield surface for beam 2D element, based on the elastic-perfect plastic stress-strain relationship, under the Von Mises yield criterion and the hypothesis of equivalence in strain of the Continuum Damage Mechanics is presented. Interaction of bending moment, axial force, shear force and damage of material are taken into account for determining a single-equation for the yield surface valid for the Navier-Bernoulli beam model with a rectangular cross section. The material yield surface decreases as damage of the beam cross section increases, as expected.

Basic assumptions, such as the following, have been taken into account for determine the yield function for beam 2D element.

- Material nonlinearity is simulated by the formation of plastic zones of zero length at the ends of the each beam element.
- The effect of strain hardening is not considered.
- For the plastic behaviour, von Mises yield criterion and associated flow rule are adopted.
- Damage (D) is isotropic and, as plasticity, it is supposed concentrated in the beam ends.
- For simplicity, all expressions are only applicable for rectangular cross section of base (b) and high (h).

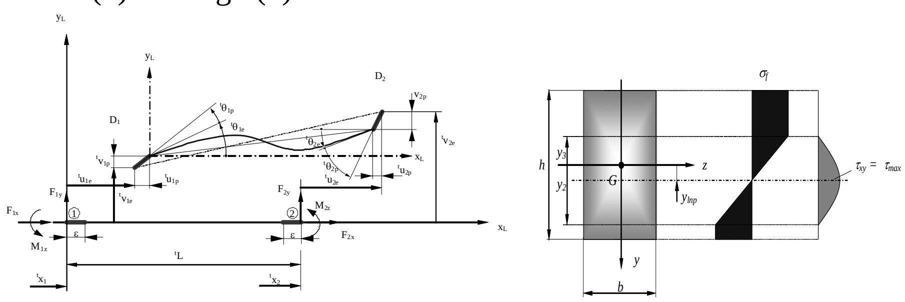

Fig.1. Beam element with elasto-plastic displacement at the end of the element and damage

We concluded that the yield function (Y_{MNVd}) for the 2D beam element taking into account the both the effects of damage and stresses due to axial and shear forces and bending moment is give by: $Y_{MNVd} = \frac{|M_z|}{M_p} + \left(\frac{N_x}{N_p}\right)^2 \frac{1}{(1-D)} + \frac{1}{3}\left(\frac{V_y}{V_p}\right)^2 \frac{1}{(1-D)^3} - (1-D) = 0$.

This expression can be used for determining the elasto-plastic stiffness matrix of the beam element and therefore the possibility for the structural analysis of frames considering damage material i.e. $dF = K^{ep} du^{ep}$ where $\boldsymbol{dF}$ is the stress vector at each beam end, K^{ep} is the elastic-plastic degradation stiffness matrix and du^{ep} is the vector of elasto-plastic displacement at the ends of the element. The yielding surfaces for a damaged material are shown in figure 2. Notice that the yield surface decreases as damage of the cross section increases.

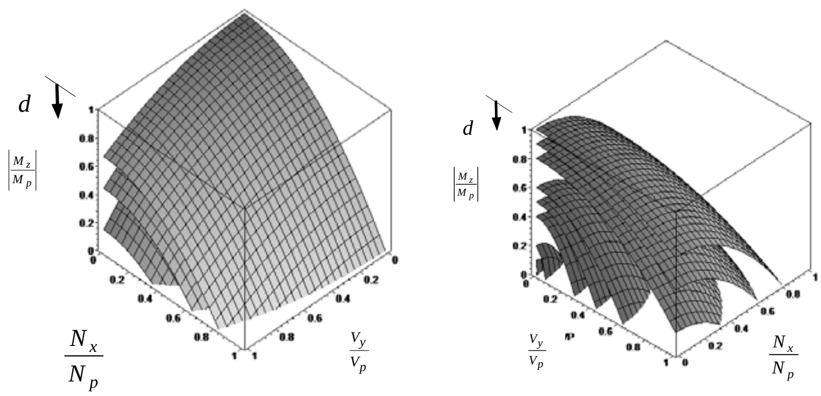

Fig.2. Yield surface for rectangular cross section depending on the stresses and damage

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