A method for identifying a spacewise dependent heat source under stochastic noise interference

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ABSTRACT

We consider the statistical inverse problem of estimating a spacewise dependent heat source in the time-dependent heat equation using the usual conditions of the direct problem and information from one supplementary temperature measurement at a given single instant of time. There are several important applications of such a model like finding a pollution source intensity given the measurements of the pollutant concentrations at a later time or designing the final state in melting and freezing processes. Since it is assumed that the data is perturbed by stochastic noise, the approach we consider is based on a statistical method suggested by Bissantz, Hohage and Munk in [1]. Due to the instability of the problem, a Tikhonov regularization is used to estimate the heat source. Using properties of the operator describing our problem (see [2]) we derive convergence results of the method by employing results from [3]. To illustrate the results by numerical experiments we discretize the continuous problem using a finite element method and the experiments show that the theoretical rates of convergence match the rates exhibited by our numerical results.

REFERENCES

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