8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Numerical resolution of a two fluid-two pressure model by a relaxation approach

*** Annalisa Ambroso[1], Christophe Chalons[1,2], Frédéric Coquel[3] and Thomas Galié[1]**

[1] DEN/DANS/DM2S/SFME
CEA Saclay
F-91191 Gif-sur-Yvette, France
annalisa.ambroso@cea.fr;
thomas.galie@cea.fr

[2] Université Paris 7-Denis Diderot et UMR 7598 Laboratoire Jacques-Louis Lions,
F-75005 Paris
France
chalons@math.jussieu.fr

[3] CNRS and Université Pierre et Marie Curie-Paris6, UMR 7598
F-75005 Paris France
coquel@ann.jussieu.fr

Key Words: *multiphase flows, two fluid-two pressure model, relaxation*

ABSTRACT

In recent years, a two fluid-two pressure approach for the modelling and computation of two-phase flows has gained interest. The basic set of equations was formulated in [1], [9], [7], [6]. In one space dimension and in the absence of source terms, they read, for $t > 0$ and $x \in \mathbb{R}$,

$$\begin{cases} \partial_t \alpha_1 + u_I \partial_x \alpha_1 = 0 \, , \\ \partial_t(\alpha_1 \rho_1) + \partial_x(\alpha_1 \rho_1 u_1) = 0 \, , \\ \partial_t(\alpha_1 \rho_1 u_1) + \partial_x(\alpha_1 \rho_1 u_1^2 + \alpha_1 p_1(\rho_1)) - p_I \partial_x \alpha_1 = 0 \, , \\ \partial_t(\alpha_2 \rho_2) + \partial_x(\alpha_2 \rho_2 u_2) = 0 \, , \\ \partial_t(\alpha_2 \rho_2 u_2) + \partial_x(\alpha_2 \rho_2 u_2^2 + \alpha_2 p_2(\rho_2)) - p_I \partial_x \alpha_2 = 0 \, , \end{cases} \tag{1}$$

where α_1, ρ_1 and u_1 (respectively $\alpha_2 = 1 - \alpha_1$, ρ_2 and u_2) represent the volume fraction, the density and the velocity of the phase 1 (resp. of the phase 2). The pressure laws p_k, $k = 1, 2$ are given smooth functions such that $p_k'(\rho_k) > 0$, $\lim_{\rho_k \to 0} p_k(\rho_k) = 0$ and $\lim_{\rho_k \to \infty} p_k(\rho_k) = +\infty$.
The system requires the definition of two closures, namely the interfacial velocity u_I and interfacial pressure p_I. We choose to set

$$u_I = u_2, \;\; p_I = p_1(\rho_1). \tag{2}$$

This choice was proposed in [1]. We refer the reader to [5] for a comprehensive study of general closure laws concerning u_I and p_I. Relations (2) ensure that the void fraction is transported by a pure contact discontinuity and allow to give sense to the system even if it can not be written in a conservative form.

We propose here a numerical scheme for the approxiamation of the solutions of this system based on a relaxation approach [2], [3], [4], [8].
The starting point is that the structure of System (1) is the one of two Euler systems coupled by the wave u_I. Indeed it is on this wave that the non-conservative products $p_I \partial_x \alpha_k$, $k = 1, 2$, play a role. The relaxation scheme is thus constructed in a way to preserve in the best possible way the contact

discontinuity u_I and this is achieved by a particular treatment of non-conservative products.

We will prove that the proposed Riemann solver is conservative for the mass of each phase and for the total momentum and that it captures exactly the u_I-contacts discontinuities.
Actually, in a classical way, when we consider the whole resolution scheme the last property can not be guaranteed for moving contacts due to the averaging procedure. It still holds true for stationary contacts as well as the conservation properties.
We will illustrate the behavior of this scheme by different test cases.

Acknowledgements

This work falls within the scope of a joint research program on the coupling of multiphase flow models between CEA-Saclay and Laboratoire Jacques-Louis Lions. It was partially supported by the NEPTUNE project, funded by CEA, EDF, IRSN and AREVA-NP.

References

[1] M.R. BAER AND J.W. NUNZIATO, *A two phase mixture theory for the deflagration to detonation (DDT) transition in reactive granular materials.* Int. J. Multiphase Flows, Vol. 12(6), (1986), pp. 861-889.

[2] C. CHALONS AND F. COQUEL, *Navier-Stokes equations with several independent pressure laws and explicit predictor-corrector schemes*, Numer. Math., Vol. 101(3), (2005), pp. 451–478.

[3] C. CHALONS AND J.F. COULOMBEL, *Relaxation approximation of the Euler equations*, submitted

[4] F. COQUEL, E. GODLEWSKI, A. IN, B. PERTHAME AND P. RASCLE, *Some new Godunov and relaxation methods for two phase flows*, Proceedings of the International Conference on Godunov methods : theory and applications, Kluwer Academic, Plenum Publisher, 2001.

[5] T. GALLOUËT, J.M. HÉRARD AND N. SEGUIN, *Numerical modeling of two-phase flows using the two-fluid two-pressure approach*, Mathematical Models and Methods in Applied Sciences (M3AS), Vol. 14(5), (2004), pp. 663–700.

[6] S. GAVRILYUK AND R. SAUREL, *Mathematical and numerical modeling of two-phase compressible flows with micro-inertia.*J. Comput. Phys., Vol. 175, (2002), pp. 326-360.

[7] J. GLIMM, D. SALTZ, AND D. H. SHARP, *Two phase flow modelling of a fluid mixing layer.*J. Fluid Mech., Vol. 378, (1999), pp. 119-143.

[8] S. JIN AND Z. XIN, *The relaxation schemes for systems of conservation laws in arbitrary space dimensions*, Comm. Pure Appl. Math., Vol. 48, (1995), pp. 235–276.

[9] A.K. KAPILA, S.F. SON, J.B. BDZIL, R. MENIKOFF AND D.S. STEWART, *Two phase modeling of DDT: structure of the velocity-relaxation zone.* Phys. Fluids, Vol. 9(12), (1997), pp. 3885-3897.