

## AN ANALYSIS OF FLUID-STRUCTURE METHODS WITH FICTITIOUS DOMAINS USING SPECTRAL/HP ELEMENTS

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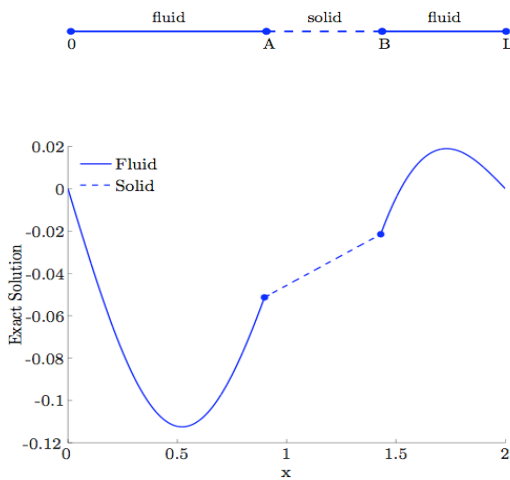
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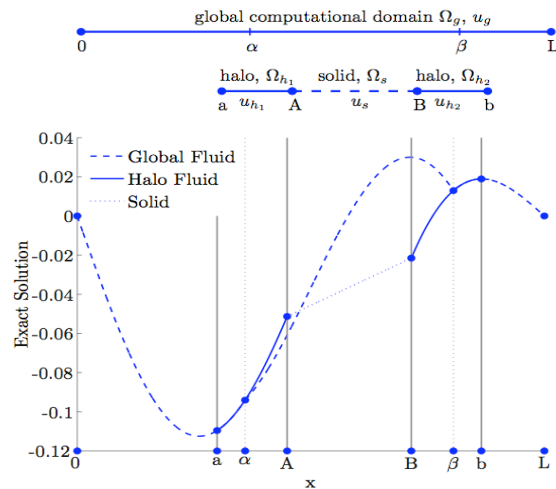
### ABSTRACT

The aim of this work was to investigate if some of the overlapping methods with fictitious fluid domains that have been proposed in the literature, are suitable to use in a spectral/hp framework. Some amendments and extensions were made where required in order to allow the approaches to deal with the necessary interface conditions. A one-dimensional problem was defined to investigate for each method if it could benefit from the exponential p-convergence that is typical for the spectral/hp elements.

The test case consisted of a fluid-structure interaction problem that was represented by a one-dimensional fluid domain with an immersed solid domain (**Fig 1**). Both domains are described by the Poisson equation with different values for the coefficients ( $\alpha_f=1$ ,  $\alpha_s=5$ ) and forcing functions ( $f_f=\sin(\pi x)$ ,  $f_s=0$ ) for the fluid and solid respectively. The analytic solution is shown in **Fig 1**.



**Fig 1:** *Problem definition: Fluid domain the with embedded solid (top) and the exact solution (bottom)*

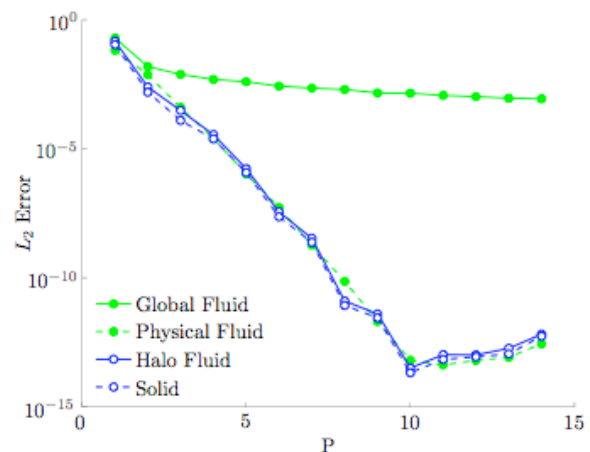


**Fig 2:** *Fluid and solid domains for a Fat Boundary Method (top) and the corresponding solutions (bottom)*

The first approach considered was the fictitious domain approach for spectral/ elements as proposed by Dong et al. [2]. The method considers a fluid domain with an overlapping solid domain and the coupling between the domains is established by a Lagrange multiplier (across the entire solid domain or only at its boundary). Both choices for the Lagrange multiplier failed to give exponential p-convergence and do not benefit from the advantages of spectral/hp elements, (which does not rule out its use for lower order element methods as demonstrated by Van Loon et al. [3]).

The second approach considered is the Finite Cell Method [4], which was used to model voids in solid plates. This method eliminates the contributions of the fictitious part of the fluid domain by pre-multiplying a part of the element matrix by some small value. When all boundary integrals are resolved correctly this method leads to exponential convergence for our test problem.

Finally, we evaluated an explicit and implicit version of the Fat Boundary Method [5], which introduces a halo fluid around the solid (**Fig 2**). The linear finite element method presented in the literature was extended in a spectral/hp framework, which leads to a typical solution (**Fig 2**). The interface conditions allow the fictitious part of the global fluid to behave differently from the exact solution, as illustrated by the convergence plot (**Fig 3**).



**Fig 3:** *p*-convergence of Fat Bound. Meth.

The simplified models in this work already provide a good understanding of the subtleties that make an overlapping method successful for spectral/hp methods or not. At present two-dimensional versions of these models are investigated for a more solid proof that methods without any alignment between the solid and the fluid have a future in (bio)mechanical engineering.

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