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Bubble stabilized discontinuous Galerkin method using a symmetric formulation for second order elliptic problems. Part II: Stokes' problem, time dependent problems and numerical aspects.

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ABSTRACT

Drawing on ideas from earlier work on the nonsymmetric DG formulation for elliptic problems (Girault et al. [5], Larson and Niklasson [4], Brezzi and Marini [1]) Burman and Stamm recently showed that a symmetric DG method using a piecewise affine approximation space, enriched by nonconforming quadratic bubbles, is stable and optimally convergent without interior penalty term [2,3]. This unstabilized approach has some interesting properties: the method enjoys enhanced local mass conservation unperturbed by any numerical parameters and the system is adjoint consistent allowing for optimal convergence of the error in the L^2–norm. On the other hand, the system resulting from the proposed discretisation is not positive definite, leading to a more complex analysis. Indeed standard coercivity arguments fail and an argument involving an inf-sup condition must be applied also for the classical Poisson problem.

In this talk we will show how the proposed method can be extended to more complex problems in fluid mechanics with special focus on the heat equation and Stokes' problem.

For the parabolic model problem we show that in spite of the presence of negative eigenvalues a standard backward Euler time discretization scheme leads to a stable and optimally convergent method. Stability holds under a non restrictive inverse parabolic CFL–condition, namely that $h^2/\delta t$ is small enough where we denote by h the spatial and by δt the temporal discretization parameter. We will also discuss the case of other A-stable time-discretization schemes.

For Stokes' problem on the other hand we investigate what pressure spaces may be used in order for the problem to be uniformly wellposed. We propose to combine the bubble enriched space for the velocities with the space of discontinuous piecewise constant functions or the space of continuous affine approximations for the pressure. We will discuss how these choices of pressure spaces relates to classical methods indicating in each case how the inf-sup condition may be proved. Moreover we

discuss the approximation properties of both variants and review the conservation properties of the resulting method.

Some numerical examples illustrating the theory will be given for the two model-problems discussed.

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