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COMPUTATIONAL ASPECTS IN THE STUDY OF A MODEL FOR THE PEDESTRIANS-INDUCED LATERAL VIBRATIONS OF FOOTBRIDGES

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ABSTRACT

We study the phenomenon of synchronization of the pedestrians motion with the natural modes of slender footbridges [1], which attracted the attention of researchers after the Millennium Bridge well known event [2].

We improve a bridge-pedestrians model developed by Strogatz & Abrams [2, 3] by taking into account also the possibility of synchronization between pedestrians, in addition to the bridge-pedestrian possible synchronization. The model is governed by ordinary nonlinear differential equations (ODEs), i.e., it is a continuous-time model, and it requires extensive numerical simulations to be utilized in practice. Our approach is therefore computational, by means of a self-made code which joins traditional numerical methods (i.e., explicit Runge-Kutta method for integrating systems of ODEs) with self-developed algorithms aimed at capturing the main dynamical aspects. The goal is to better understand the underlying mechanical phenomena, and to increase the agreement of the model results to the effective behaviour of the walkers, as observed during the Millennium Bridge's opening day and during Arup's tests on site [4].

Our analyses confirm that the model catches very well the nonlinear nature of the problem in terms of dynamic response of the bridge: the oscillations are small until a critical number of walking pedestrians N_c and then, due to the synchronizations, they increase rapidly and uncontrollably until a final threshold. The critical number of pedestrians, which causes the vibrations to increase suddenly to unacceptable levels, is of practical engineering interest and its correct prediction is the final aim of all the theoretical modelling studies.

We develop a numerical technique to fit the amplitude of oscillations versus time curve, obtained by numerical simulations, by a piecewise linear averaging curve. Actually, we use three straight segments, which is sufficient for our purposes. In fact, by observing Abrams' model results, which are in good agreement with the experimental curves of bridge vibration amplitude and pedestrian synchronization vs time on the Millennium Bridge, we can clearly identify three different ranges, in each of them the data points have an approximately linear

trend. This three-linear regression is not so trivial, because it is, in fact, a nonlinear regression problem due to the two unknown intersection points of the fitting segments. In particular, in this work we are interested in finding the intersection point between the first and the second set of data with linear trend, as it corresponds to a reliable and automatic estimate of N_c. We do that in two different and independent ways: at first, by using the simplex search method of Nelder and Mead [5, 6] and than we obtain the same result by minimizing data misfit in the least-squares sense with proper constraints through a derivative-based search for the solution [7].

Our computational method is able to detect automatically the number of pedestrians which trigger the synchronization, i.e., the critical threshold which points out the simultaneous onset of bridge instability and of crowd synchronization. This allows us to perform a wide set of simulations with an acceptable CPU time, thus deriving statistically reliable considerations: for each tested case the results, in terms of crowd's critical size, are randomly distributed approximately according to a Gaussian and we are able to detect its mean value and its standard deviation.

The numerical methods we have applied permit to investigate the effects on the system dynamics of all the technical parameters and human crowding factors, thus obtaining a quite good understanding of the model and of the underlying physical phenomenon. Moreover our computational solutions for Strogatz and Abrams' extended model may prove useful for estimating the damping needed to stabilize other exceptionally crowded footbridges against synchronous lateral excitation by pedestrians, or for designing other methods aimed at eliminating the phenomenon in real structures.

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