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A CONVERGENT EXPLICIT SCHEME FOR SIMULATING CONVECTION-DIFFUSION PHENOMENA

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ABSTRACT

The efficient solution of problems combining both diffusive and convective phenomena, is a crucial step to handle numerical simulations in countless technological applications, in particular those involving fluid flow. This work deals with numerical methods based on variational formulations such as the finite element method, to solve convection-diffusion equations.

In this framework, one of the first techniques employed to model convection was the so-called Lesaint-Raviart method (cf. [6]). However a little later relevant contributions to the time-dependent case incorporating diffusion arose, as it is well reported in [4]. In this respect we should quote the pioneering upwinding schemes due to Tabata and collaborators (see e.g. [8] and [1]).

Since the mid-eighties, the most widespread manner to deal with dominant convection has been the use of stabilizing procedures based on the space mesh parameter, among which the streamline upwind Petrov-Galerkin (SUPG) technique introduced by Brooks & Hughes (cf. [2]) is one of the most popular. However, as far as time time-dependent problems are concerned, it turns out that the time step plays a better stabilizing role, provided a formulation well suited to the equations to be solved is employed. A good illustration of this assertion in the case of the time-dependent Navier-Stokes equations can be found in Codina & Zienkiewicz [3] or yet in [7].

The authors intend to give a contribution in this direction, in the case of the convection-diffusion equations, discretized in space with piecewise linear finite elements, combined with a non standard explicit backward Euler scheme for the time integration, and a standard Galerkin approach. The method is particularly performing in the case of largely dominant convection.

Our main theoretical result states that the numerical solution is stable in the maximum norm in both space and time, and even convergent under certain angle conditions, provided that roughly the time step is bounded by the space mesh parameter multiplied by a mesh-independent constant that we specify. As the authors should clarify, the scheme studied in this paper follows similar principles to the one long exploited by Kawahara and collaborators, for simulating convection dominated phenomena (see e.g. [5] among other papers published by them before and later on). The originality of our contribution relies on the fact that we not only introduce a reliable scheme for any space dimension, but also exhibit rigorous conditions for it to provide converging sequences of approximations in the sense of the maximum norm.

An outline of this contribution is as follows: After recalling the problem to solve we describe the type of discretisation corresponding to the new method. More particularly we specify a weighted manner to deal with the finite element discretisation matrices, in order to obtain efficient schemes. Then stability results for our method in the sense of the space and time maximum norm, applying to a non restrictive set of weights are given, and next conditions to be satisfied by the weights allowing for optimal error estimates are specified. Finally we give a particularly representative summary of the numerical results that we have obtained so far with our new scheme. The numerical examples include comparative convergence tests with other schemes.

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