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COUPLED ANALYSIS OF REACTOR VESSEL BY DOMAIN DECOMPOSITION METHODS

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ABSTRACT

The coupled hydro-thermo-mechanical analysis of prestressed concrete reactor vessels is a very demanding task especially if the three dimensional model has to be used. Except of difficulties with selection of an appropriate material model and suitable model tuning, there are also severe problems with solution of systems of equations which are obtained after application of the finite element method. Material models and a suitable computational framework for coupled problems can be found in reference [1].

Fine three dimensional finite element meshes lead to large systems of equations which are characterized by a large number of unknowns and a huge bandwidth (distance of the farthest nonzero off-diagonal entry in the row/column from the diagonal). In the case of hydro-thermo-mechanical analysis, the situation is serious because there are three displacements, temperature and two pore pressures at each node. Such large systems of equations cannot be efficiently solved on a single-processor computer due to lack of RAM and the number of necessary arithmetic operations.

Iterative methods are recommended for large systems of equations but it has to be taken into account that coupled problems lead in many cases to systems of equations with significantly different matrix entries. It means that the matrices are ill-conditioned and the iterative methods perform many iterations in order to obtain an acceptable norm of residuum.

Finally, the analysis of a vessel is nonstationary and the system of equations has to be solved at each time step. In the case of modelling of the whole life of the vessel, thousands of time steps has to be used. Time elapsed for such analysis then grows to several weeks.

Difficulties mentioned above lead to application of an efficient solver. Domain decomposition methods applied on parallel computers are an example of an efficient solver. The coupled problem is solved by the Schur complement method because it enables to solve symmetric as well as nonsymmetric systems. Some material models of coupled problems may result in nonsymmetric systems of equations.

The Schur complement method is based on a special ordering of unknowns where the internal unknowns are ordered first and the interface unknowns last. The internal unknowns are eliminated and the reduced system contains only the interface unknowns. The reduced system is usually solved by an iterative method. In the case of a symmetric system of equations, the conjugate gradient method is usually used while in the case of nonsymmetric systems, the bi-conjugate gradient method is used. Combination of the direct method for elimination of the internal unknowns and the iterative method for solution of the reduced problem results in an efficient algorithm. The method is simply parallelizable because the elimination of the internal unknowns can be done on each subdomain independently of other subdomains. More details can be found in reference [2].

In order to obtain a fast algorithm, the elimination of the internal unknowns is done by the sparse direct solver based on a quotient graph and an approximate minimum degree algorithm. The reduced system of equations is solved by the conjugate gradient method where the matrix-vector multiplication is distributed. It means that the Schur complements of particular subdomains are located on processors and the multiplication is done in parallel. The matrix of the reduced system is not assembled.

33 years of life of existing prestressed concrete reactor vessel is modelled. The time step is in hours, at most one day, with respect to mechanical and thermal loading. Two dimensional axisymmetric and three dimensional models are used for the analysis. Influence of the application of the sparse direct solver as well as influence of various numbers of subdomains on total efficiency of the algorithm are studied. Selected parts of the reactor vessel response will be shown.

The proposed strategy based on combination of a direct solver and iterative one is suitable for very large systems of equations. For such systems, it cannot be compared with pure direct or iterative solvers. Direct solvers are unable to solve large matrices and iterative solvers spend a lot of time during many iterations necessary for prescribed accuracy. Reduced twodimensional model (coarser mesh was generated) of the vessel was solved by a direct solver and it took several weeks while the proposed strategy applied on 6 processors took only 2 days.

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