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INVERSE SCATTERING OF AN ARBITRARILY SHAPED BURIED SCATTERER WITH CONDUCTIVE BOUNDARY CONDITION

F. Yaman

Istanbul Technical University, Electrical and Electronics Engineering Faculty, 34469 Maslak, Istanbul, Turkey
yamanf@itu.edu.tr and http://atlas.cc.itu.edu.tr/ yamanf/

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ABSTRACT

In this study, an integral equation based method for arbitrarily shaped cylindrical objects that have conductive boundary condition on their surface and are buried in arbitrarily shaped cylindrical dielectrics is presented. The aim of the direct scattering problem is to obtain the far field in the case of time harmonic plane wave incidence. The inverse problem considered here is the reconstruction of the *conductivity function* of the buried scatterer from meausurements of the far field for one incident wave, assuming that the shapes of all scatterers are known. Both for the direct and the inverse problems potential approach is used to obtain a system of boundary integral equations which are numerically solved by Nyström method and Tikhonov regularization is applied to the first kind of integral equations.

Let $D_0 \subset \mathbb{R}^2$ be a bounded medium closured by a smooth curve Γ_0 and D_1 denotes a doubly connected bounded medium with a smooth boundary ∂D_1 consists of an interior boundary Γ_0 and an exterior boundary Γ_1 such that $\partial D_1 = \Gamma_0 \cup \Gamma_1$ and $\Gamma_0 \cap \Gamma_1 = \emptyset$. The unbounded domain D_2 is connected with the boundary Γ_1 to the domain D_1. We shall denote by ν_0 the unit normal of the boundary Γ_0 directed into exterior of D_0 and by ν_1 the unit normal of the boundary Γ_1 directed into exterior of D_1. The direct scattering problem is to find the far field u_∞ where the total fields u_j have to satisfy Helmholtz equations in the corresponding domains,

$$\triangle u_j + k_j^2 u_j = 0 \quad \text{in } D_j \ , \quad j = 0, 1, 2$$

with complex wave numbers k_j, and additionally transmission and conductive boundary conditions over the boundaries,

$$u_1 = u_2 \quad \text{and} \quad \frac{\partial u_2}{\partial \nu_1} = \frac{\partial u_1}{\partial \nu_1} \quad \text{on } \Gamma_1 \ , \qquad u_0 = u_1 \quad \text{and} \quad \frac{\partial u_1}{\partial \nu_0} - \frac{\partial u_0}{\partial \nu_0} = \lambda u_1 \quad \text{on } \Gamma_0 \ .$$

under the Sommerfeld radiation condition. Here, λ is the so called conductivity function. According to our approach we try to find a solution of the direct problem by representing the fields in the form of combined single- and double- layer potentials and we obtain a system of equations for unknown density functions using jump relations, see [1]. Once the densities are determined one can compute the far field pattern which is defined over the unit circle. The noisy data is obtained by adding random errors pointwise to the u_∞.

For the solution of the inverse problem in order to avoid *inverse crime* we seek all the fields in the form of single-layer potentials. Firstly, Tikhonov regularization is applied to the ill-posed far field equation to reconstruct a density function, then by using this density the total field u_2 is computed. Furthermore, transmission conditions allow us to obtain the total field u_1 via second Tikhonov regularization. As a following step, one can compute the total field u_0 by using the continuity of the total fields over the boundary Γ_0 [2,3]. Finally, the conductivity function λ, can be determined from the equation

$$\frac{\partial u_1}{\partial \nu_0} - \frac{\partial u_0}{\partial \nu_0} = \lambda u_1 \quad \text{on } \Gamma_0 \, ,$$

where possible zeros in the denominator will be taken care off by least squares regularization.

In numerical results, ϕ_0 denotes the illumination angle and n is the number of discretization points. As a first numerical example (left), an elliptic cylinder with $\lambda = 1 + \sin^3 x + i \sin x$ is buried in a circular cylinder where $k_0 = 3, k_1 = 2, k_2 = 1, \phi_0 = 0$ and $n = 128$ are chosen. In the second example (right), an elliptic cylinder having $\lambda = \sin^4 x/2 + i \cos^4 x/2$ is buried in a peanut shaped cylinder. Parameters $k_0 = 2, k_1 = 4, k_2 = 2, \phi_0 = 180$ and $n = 64$ are chosen.

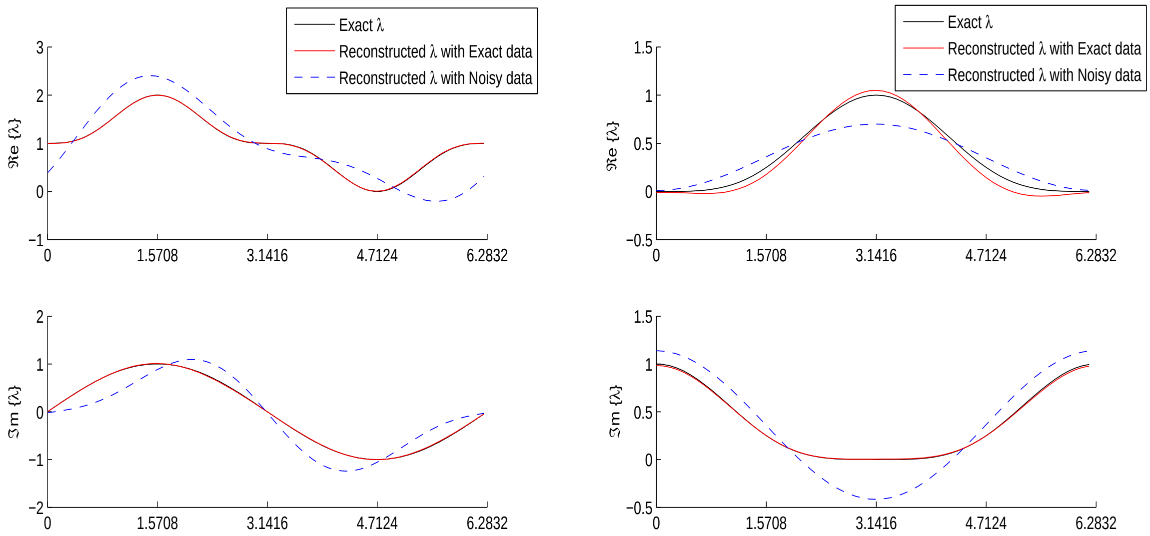

Figure 1: Reconstruction of the conductivity function without noise and with $\%3$ noise

In conclusion, the proposed method gives satisfactory reconstructions by using exact data with smooth scatterers and optimum illumination angle, however reconstructions start to deteriorate when the noise level exceeds 3% - 4%.

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