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A hybrid finite volume – boundary element method (FV-BEM) for the numerical solution of the kinematic induction equation

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ABSTRACT

The experimental realization of dynamo excitation as well as theoretical and numerical examinations of the induction equation have shown the relevance of boundary conditions and material properties for a self-sustaining dynamo [1,2]. Laboratory (insulating) boundary conditions in non-spherical geometry, in general, are treated by elaborated schemes (e.g. solving of the Laplace equation in an extended domain, [3]) or by simplifying approximations (pseudo vacuum, vanishing tangential field).

Rather precise results for the numerical solution of the kinematic induction equation are provided by the integral equation approach which also considers insulating boundary conditions exactly [4,5]. However, the application possibilities are limited because of enormous computational resources that are required by this method. In particular in the context of inverse problems – e.g. estimation of the velocity structure from measurements of induced electric and/or magnetic fields – solving the forward model turned out to be the most time consuming part.

Therefore, a more flexible approach utilizing a local discretization like the constraint transport (CT) method as a well known realization of a finite volume scheme (FV) is adopted for solving the induction equation. The CT-scheme ensures a fast, robust and accurate solution of the kinematic dynamo problem and intrinsically maintains the solenoidal character of the magnetic field [6]. Within the framework of the finite volume scheme insulator boundary conditions are treated by a modified integral equation procedure, commonly known as the boundary element method (BEM) [7]. On the boundaries the magnetic flux density $\boldsymbol{B}$ can be expressed as the gradient of a scalar potential $\boldsymbol{B} = -\nabla\Phi$, where Φ is described by a Laplace equation: $\Delta\Phi = 0$. Integrating the Laplace equation only on the boundaries requires less computational power than the full integral equation approach. Nevertheless, the difficulties of the non-local character of insulating boundary conditions can still be overcome and the direct computation of the magnetic field next to an insulator is possible. Combining both methods in a hybrid FV-BEM scheme offers the flexibility of a local discretization with a stringent treatment of insulating magnetic boundary conditions in almost arbitrary geometries [8].

The fast and easy to handle algorithm exhibits further advantages when considering spatial varying material properties like electrical conductivity of container walls or localized high-permeability material. Discontinuities of material coefficients are treated within the framework of the FV-BEM scheme under

utilization of appropriate averaging procedures for the permeability/conductivity on the contact interfaces which ensures that the corresponding jump conditions for the magnetic flux density $\boldsymbol{B}$ and/or the electric field $\boldsymbol{E}$ are fulfilled [9].

Several test computations with prescribed α-effect or velocity distribution reproduce well known key results and demonstrate the applicability and reliability of the approach. Future examinations are intended to understand the behavior of the VKS-dynamo experiment where the field producing flow is driven by ferrous propellers and the dynamo mechanism probably is strongly influenced by this high permeability material. Indeed, preliminary results including ferromagnetic material inside the computational domain show a certain reduction in the critical magnetic Reynolds number $\mathrm{Rm} = \mathcal{LV}/\eta$ but, up till now, they cannot explain the dominating axisymmetric field mode that is observed in the experiment. Other applications of the hybrid scheme may also be important for the evaluation of forthcoming dynamo experiments for which a precise knowledge of the critical magnetic Reynolds number is essential. Finally, the method is of interest for the treatment of inverse problems in industrial applications like estimation of the liquid metal flow in continuous casting from measurements of induced magnetic fields via contactless inductive flow tomography [10].

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