

## Efficient (Error Controlled) Calculation of Non-Linear Problems by the Boundary Element Method

\* Klaus Thoeni<sup>1</sup> and Gernot Beer<sup>2</sup>

<sup>1</sup> Institute of Structural Analysis  
Graz University of Technology  
Lessingstraße 25/II, Austria  
thoeni@tugraz.at

<sup>2</sup> Institute of Structural Analysis  
Graz University of Technology  
Lessingstraße 25/II, Austria  
gernot.beer@tugraz.at

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### ABSTRACT

Previous publications have shown that the boundary element method (BEM) can be used to solve non-linear problems, e.g. Telles [7], Venturini [8]. By using this method, usually only the boundary of the problem has to be discretized. Thus, the dimension of the problem is reduced by one. Regarding mesh generation, the dimension of the system of equations, data storage, and post-processing, this is considered a significant advantage.

When dealing with non-linear problems, not only boundary integrals but also domain integrals arise. In the standard 2d or 3d approach, a mesh of area or volume cells is used respectively for the evaluation of the domain integrals (see Beer [1], Gao and Davies [3]). Alternatives to this technique are the Dual and Multiple Reciprocity Methods that transfer the volume integral into a boundary integral (see Nowak and Neves [5]). However, these methods have significant drawbacks especially when dealing with infinite domains. Many other methods for solving the domain integral exist, like the radial integration method published by Gao [2] or the pure BEM approach published by Noronha et al. [4]. An in-depth review of currently used procedures for non-linear analysis with the BEM is available in Gao [2].

In this work the standard cell method is used. By using this method the main advantage of the BEM with respect to mesh generation is lost because cells have to be generated by the user, either in the whole domain or in parts of the domain that are expected to behave non-linearly. Moreover, the evaluation of the internal results and the integration over the cells are rather time consuming. However, the size of the system of equations does not depend on the domain discretization, which means that no additional degrees of freedom will be introduced using internal cells. In addition, internal cells can be generated automatically (see Ribeiro et al. [6]). But as we will show the accuracy of the results strongly depends on the size and type of the cells.

In this paper we present a new method which automatically controls the domain discretization error. The new method uses an automatic algorithm to refine the domain cells, based on a local error estimator. As mentioned above for the evaluation of the domain integral, which considers the influence of the

initial stress arising from the non-linear behaviour, a cell mesh is used. The area and volume cells used in the formulation are of type  $C^0$ , and therefore they can not reproduce the expected smooth initial stress field in the non-linear zone. This discrepancy between the cells is used to estimate the error and to refine the domain cells where it is needed. The results show that the method is general and can be used to solve 2-dimensional as well as 3-dimensional problems. Moreover, it is not limited to special non-linear models. We will demonstrate the applicability and the accuracy of this new method on some elasto-plastic problems.

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