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PROBABILISTIC CHARACTERISATION OF SURFACE IMPERFECTIONS IN PIEZOELECTRIC CERAMICS

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ABSTRACT

Piezoelectric ceramics are materials that exhibit relatively large deformations through application of an electric field and vice versa. This electromechanical coupling makes piezoelectrics suitable for many applications, such as MEMS (micro electromechanical systems). The brittleness of piezoelectric ceramics makes the materials, however, sensitive to damage. An accurate and robust numerical model to asses this damage is an indispensable tool for increasing the reliability of piezoelectric systems.

Fracture of piezoelectric ceramics such as PZT (Lead Zirconate Titanate) has been studied both experimentally and numerically [1,2]. Focus has been on the determination of a fracture criterion that correctly mimics the influence of an electric field on the ultimate load. In Ref. [2] it is demonstrated that a fracture criterion based on the mechanical fracture toughness is in good agreement with experimental observations for specimens with an initial crack such as the three-point bending test (Figure 1, left). In practice, a manufactured pre-crack is not present in a piezoelectric component. Fracture initiation will in that case occur due to a-priori unknown imperfections at the specimen boundary. Since the position and size of these imperfections can vary significantly, a large spread in possible fracture paths and fracture loads is to be expected (Figure 1, right). A convenient method for analysing such a problem is to describe the fracture strength on the surface by a stationary random field. Stochastic finite element methods [3] can then be used to quantify statistical properties of the response.

For the construction of the random field for the fracture criterion, it is important that the imperfections causing crack initiation are identified correctly. Here, two types of boundary imperfections are considered as being dominant for fracture initiation. First the surface roughness caused by machining of the component is considered. Second, the presence of unaligned polarisation domains is taken into account. These unaligned domains are a result of an imperfect polarisation procedure. Abrupt changes in polarisation magnitude and direction give rise to stress concentrations and are therefore identified as weak spots on the surface.

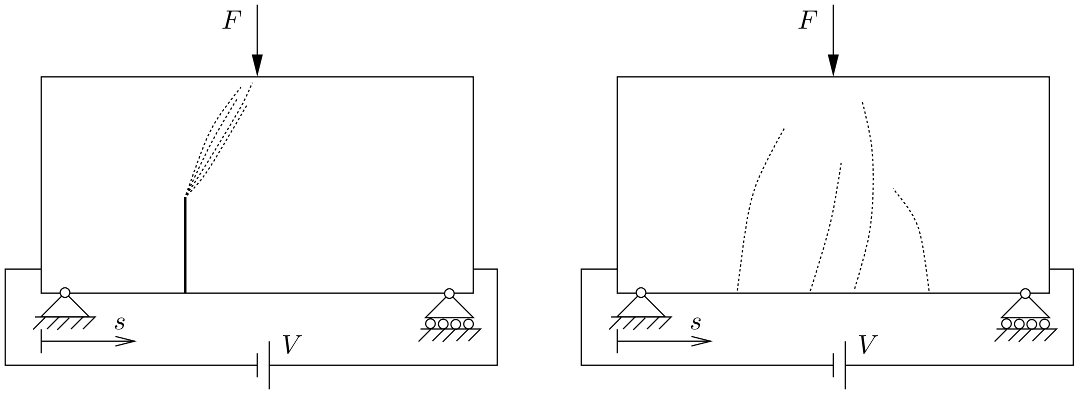

Figure 1: The three-point bending bending test as considered in Ref. [2] with an off-centred initial crack (left) and a similar specimen without initial crack (right). In contrast to the case with an initial crack, the crack nucleation positions are a-priori unknown if no initial crack is present.

The moving window technique [4] is used to construct the random field for the fracture strength. Given the fracture strength f_{ult} at N discrete points on the considered surface, the moving window technique approximates its mean and spatial covariance function respectively as

$$\mu_{f_{\mathrm{ult}}} = \sum_{i=1}^{N} f_{\mathrm{ult}}(s_i) \quad \text{and} \quad \Sigma_{f_{\mathrm{ult}}}(\Delta s) = \frac{1}{N-1} \sum_{i=1}^{N} \left(f_{\mathrm{ult}}(s_i) - \mu_{f_{\mathrm{ult}}} \right) \left(f_{\mathrm{ult}}(s_i + \Delta s) - \mu_{f_{\mathrm{ult}}} \right). \quad (1)$$

Using this statistical information in combination with an assumed probability distribution (e.g. lognormal), a Karhunen-Loeve expansion of the random field can be obtained to parametrise the random field. Alternatively, higher-order cumulants can be used to derive a more appropriate probability distribution.

Determination of a random field using the moving-window technique requires: 1) A micro structural representation of the edge. In this contribution, an edge is generated numerically using micromechanical models for the generation of the two types of imperfections mentioned above. 2) Evaluation of the ultimate strength at the N discrete points used for the moving-window technique. The ultimate strength at the discrete locations $\{s_i\}$ is computed using computational homogenisation. This implies that a finite element model capable of modelling inter- and transgranular fracture is used to determine the fracture strength at the sampling points.

The proposed method is tested for various settings of the microscopic imperfections. The influence of the window size Δs used for the moving window technique is investigated as well. The random field obtained for the fracture criterion is parametrised such that it can be used for stochastic finite element simulations.

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