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ADAPTIVE TIME-STEPPING ALGORITHM FOR STRAIN-SOFTENING MATERIALS

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ABSTRACT

The present work falls into category of damage-driven adaptive strategies. In contrast to other control mechanisms for mesh adaptivity we include not only the variations in the stress field into account but also the spatial variations in the material parameters. Such an approach is especially important in cases when the two fields exhibit similar scales of variability resulting in complex size effect behavior.

The usual damage/plasticity framework is used to formulate the extension of the standard time-integration algorithm. The refinement/coarsening strategy is controlled by the relative distance to the inelastic range at each material point. This variable controls both the time- and space- resolution of the discretization. In particular, the formulation renders: (1) general criteria for controlling the time-step, and (2) spatially defined mesh resolution control function (MRCF) reflecting the variability of arbitrary material parameters affecting both stiffness and strength.

The mesh control is based on the assumption that the used material model introduces a loading function

$$f(\boldsymbol{\varepsilon}, \boldsymbol{\kappa}, \boldsymbol{\theta}) \leq 0 \tag{1}$$

representing the transition to an inelastic zone, with $\boldsymbol{\varepsilon}, \boldsymbol{\kappa}$ and $\boldsymbol{\theta}$ denoting the spatially varying fields of strain, internal variables and material properties. Regarding a time-step $n+1$, iteration k and a quadrature point $x_I \in \Omega_I$ the loading function is calculated as

$$f^{(k)}_{I,n+1} = f(\boldsymbol{\varepsilon}^{(k)}_{I,n+1}, \boldsymbol{\kappa}_{I,n}, \boldsymbol{\theta}_I) \gtreqless 0. \tag{2}$$

that is normally followed by evaluation of trial stresses, return mapping or damage evaluation and calculation of the equilibrium residual. In the present algorithm we require that prior to this evaluation the loading function must not be violated:

$$f^{(k)}_{I,n+1} = f(\boldsymbol{\varepsilon}_{I,n} + \eta^{(k)}_I \Delta\boldsymbol{\varepsilon}^{(k)}_{I,n+1}, \boldsymbol{\kappa}_{I,n}, \boldsymbol{\theta}_I) \leq 0. \tag{3}$$

The parameter $\eta_I^{(k)}$ is introduced to adjust the time step before proceeding with the next iteration. Assuming that the material response is linear within the elastic domain and realizing that the internal variables $\boldsymbol{\kappa}$ are frozen during the predictor step equation (3) can be rewritten as:

$$f_{I,n} + \eta_I^{(k)}(f_{I,n+1}^{(k)} - f_{I,n}) \leq 0 \tag{4}$$

with $f_{I,n}$ representing the value of loading function at the last equilibrated step. Collecting the material points experiencing loading

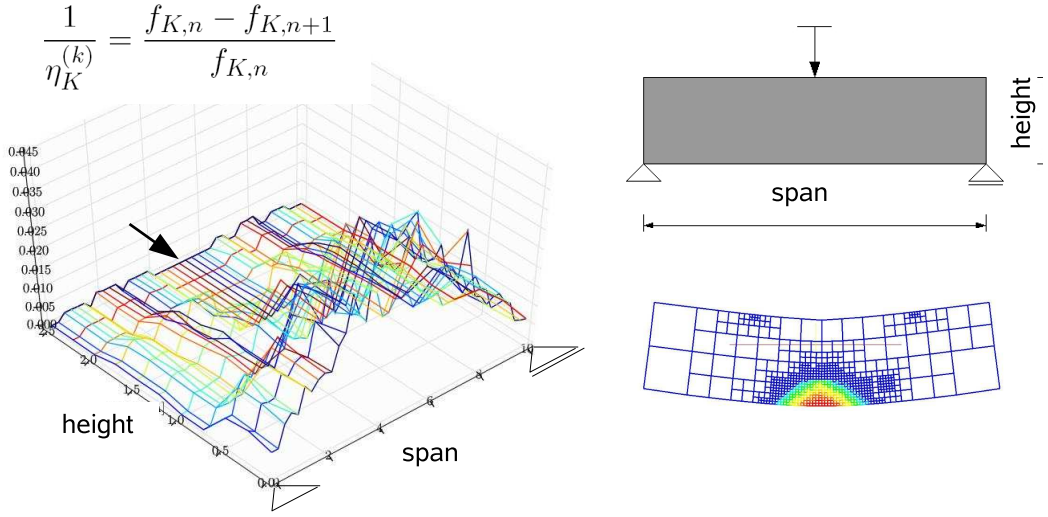
$$\Omega_K = \{I | f_{I,n} < f_{I,n+1}^{(k)}\},$$

the scaling factor adjusting the step such that equation (3) is equal to zero is obtained as

$$\eta_K^{(k)} = \frac{f_{I,n}}{f_{K,n} - f_{K,n+1}^{(k)}}, \ \forall K \in \Omega_K. \tag{5}$$

The further elaboration of η_K leads to a complex refinement/coarsening strategy connected with the scaling of the load step. The Figure exemplifies the procedure on a three-point bending test with

$$\frac{1}{\eta_K^{(k)}} = \frac{f_{K,n} - f_{K,n+1}}{f_{K,n}}$$



randomized strength. The element size is controlled by the autocorrelation length of the random strength field. In this way, the zooming into the sought displacement and stress fields is performed on demand following the goal not to miss any extreme in the spatially varying failure surface. After the onset of strain-softening in the identified and refined localization zone the unloading in other parts may lead to coarsening in other regions. The questions to be addressed using the described framework include the exchangeability and interaction of the length scales representing the variability (statistical) and the length scale of the redistribution process introduced by energetic considerations [1]. This question belongs to the general discussion on hierarchy of length scales.

REFERENCES

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