8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

A POSTERIORI ERROR ESTIMATIONS FOR FRICTIONAL CONTACT PROBLEMS APPROXIMATED BY THE EXTENDED FINITE ELEMENT METHOD

*** V. LLERAS[1], P. HILD[1] and Y. RENARD[2]**

[1] Laboratoire de Mathématiques de Besançon,
Université de Franche-Comté / CNRS UMR 6623,
16 route de Gray, 25030 Besançon, France.
vanessa.lleras@univ-fcomte.fr,
Patrick.Hild@univ-fcomte.fr

[2] Pôle de Mathématiques, INSA de Lyon,
Institut Camille Jordan, UMR CNRS 5208
20 rue Albert Einstein, 69621 Villeurbanne, France.
yves.renard@insa-lyon.fr

Key Words: *A Posteriori Error Estimates, Residuals, XFEM Method, Unilateral Contact, Coulomb Friction, Stabilized Lagrange Multiplier Method.*

ABSTRACT

The benefits of computational methods using classical finite element strategies are limited when solving problems defined over cracked domains. Indeed the mesh should be sufficiently refined around the crack tip to model the singular strain and the domain should be remeshed step by step according to the geometry of the crack propagation. To overcome these difficulties and to make the finite element methods more flexible, Moës, Dolbow and Belytschko ([12,13]) have introduced in 1999 the XFEM (eXtended Finite Element Method). The idea of XFEM consists in enriching the basis of the classical finite element method by a step function along the crack line to take into consideration the discontinuity of the displacement field accross the crack and by some non-smooth functions representing the asymptotic displacement around the crack tip. This enrichment strategy allows the use of a mesh independent of the crack geometry.

The main novelty in our work consists in taking into account the frictional contact conditions in the XFEM method (see [6],[7],[14] for existing studies) and to propose the corresponding residual estimators. Therefore we adapt to the XFEM method a stabilization technique presented by Barbosa and Hughes in [2,3,4]. We combine the XFEM approach together with the Barbosa-Hughes stabilized formulation following the ideas of [8] and [10]. The advantage of this method is that its converges whatever the intersection of the domain with the mesh is and no discrete Babuška-Brezzi inf-sup condition is needed. Besides we show that the discrete frictional contact problem admits at least a solution.

In a second time, we study a posteriori error estimators of residual type [1,15] (see [5] for another a posteriori analysis for XFEM). The finite element methods allow to approach numerically the solution $\mathbf{u}$ of a problem by a function $\mathbf{u}_h$. The question is to estimate the exact error $\|\mathbf{u} - \mathbf{u}_h\|$ with the a posteriori estimator $\eta(\mathbf{u}_h)$ which can be calculated explicitly. Thus we can evaluate the quality of the finite element computations. We use the techniques developed in [9,11] in the case of frictional contact problems approximated by a standard finite element method which we extend to the XFEM method.

REFERENCES

[1] I. Babuška and W. Rheinboldt, “Error estimates for adaptive finite element computations”, *SIAM J. Numer. Anal.*, Vol. **15**, 736–754, 1978.

[2] H. J.C. Barbosa and T.J.R. Hughes, “The finite element method with Lagrange multipliers on the boundary: circumveting the Babuška-Brezzi condition”, *Comput. Methods Appl. Mech. Engrg.*, Vol. **85**, 179–192, 1991.

[3] H. J.C. Barbosa and T.J.R. Hughes, “Boundary Lagrange multipliers in finite element methods: error analysis in natural norms”, *Numer. Math.*, Vol. **62**, 1–15, 1992.

[4] H. J.C. Barbosa and T.J.R. Hughes, “Circumveting the Babuška-Brezzi condition in mixed finite element approximations of elliptic variational inequalities”, *Comput. Methods Appl. Mech. Engrg.*, Vol. **97**, 193–210, 1992.

[5] S. Bordas and M. Duflot, “Derivative recovery and a posteriori error estimation in extended finite element methods”, *Comput. Methods Appl. Mech. Engrg.*, Vol. **196**, 3381–3399, 2007.

[6] S. Geniaut, “Approche X-FEM pour la fissuration sous contact des structures industrielles”, *Thèse de doctorat de l’école centrale de Nantes*.

[7] S. Geniaut, P. Massin and N. Moës, “A stable 3D contact formulation for cracks using XFEM”, *Revue Européenne de Mécanique Numériques, Numéro spécial: calculs avec méthodes sans maillage*, Vol. **16**, 259–275, 2007.

[8] J. Haslinger and Y. Renard, “A new fictitious domain approach inspired by the extended finite element method”, submitted.

[9] P. Hild and V. Lleras, “Residual error estimators for Coulomb friction”, submitted.

[10] P. Hild and Y. Renard, “A stabilized Lagrange multiplier method for the finite element approximation of contact problems in elastostatics”, *Prépublications du laboratoire de mathématiques de Besançon*, $n°$ **20**, 2007.

[11] V. Lleras, Thesis in preparation.

[12] N. Moës and T. Belytschko, “XFEM : nouvelles frontières pour les éléments finis.”. *Revue Européenne des éléments finis (Calcul des structures GIENS’01)*, Vol. **11**, 305–318, 2002.

[13] N. Moës, J. Dolbow and T. Belytschko, “A finite element method for crack growth without remeshing”, *Int. J. Numer. Meth. Engrg.*, Vol. **46**, 131–150, 1999.

[14] N. Moës, J. Dolbow and T. Belytschko, “An extended finite element method for modeling crack growth with frictional contact”, *Comput. Methods Appl. Mech. Engrg.*, Vol. **190**, 6825–6846, 2001.

[15] R. Verfürth, *A review of a posteriori error estimation and adaptive mesh-refinement techniques*, Wiley and Teubner, 1996.