## Discontinuous fields integration in a structured mesh using the level set method

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## ABSTRACT

Finite element analysis is now a part of engineering design. In this process, the generation of the mesh is still considered to be the greediest part in term of human time. Any method that enables to reduce mesh generation time, especially for complex geometries, has to be investigated. Moreover, if a decent mesh is available, a slight modification of the geometry should result in a slight modification of the mesh. For example, introducing a crack, a hole or inclusions in a complex mechanical part should not lead to a change in the CAD model and a complete mesh generation. With that aim in mind, researchers have introduced the idea of an *eXtended Finite Element Method* (XFEM) where additional features such as cracks or holes are defined using an implicit representation [1]. *Level set* methods enable to represent interfaces in a fixed mesh. In order to take into account the specific behavior of fields across those discontinuities, the finite element approximation of the fields is enriched in various ways in accordance with the additional feature: discontinuous enrichement for cracks, discontinuous derivatives for composite materials, asymptotic solutions at crack tips.

The XFEM approach has been investigated thoroughly in the last ten years. In this paper, we try to adress two issues that have been underway for a long time.

In his seminal paper, N. Moës [2] points out the fact that, if the level set passes too close to a node, it should be clipped onto it. In other words, the level set value should be put to zero on nodes where it is below a given treshold. This "clip to vertex" algorithm has been the cause of many discussions in the XFEM community, mainly because there is no general definition of this treshold. Here, we try to adress the same problem using a more general procedure. In our approach, we allow mesh vertices to move in the vicinity of the level set in order to maximize the volume fraction on each side of the level set, while keeping the elements undistorted. A XFEM-based r-adaptive procedure is presented that

aims to ensure that elements that are cut by the level set are split in two parts of equal volume.

The second issue we try to adress here is linked to numerical integration. In XFEM, it is required to perform numerical integration of discontinuous functions. This can only be done efficiently (i.e. using Gaussian quadrature) if integration points and their weights are defined on each side of the discontinuity. This discontinuous integration process has been successfully applied to linear simplical elements. Here, we extend that approach for level sets that have a higher order representation (e.g. in a tri-linear hexahedron). The result of our procedure is an adaptive integrator that takes as input a high order finite element with possibly several level set values at its nodes and the polynomial order of each level set, and gives as output a list of integration points together with their weights that allow to integrate exactly a function that is possibly discontinuous across the interfaces defined by the iso-zeros of the level sets. An other important capability related to XFEM is the possibility of integrating functions that are defined on the interface, in order to apply boundary conditions on the interface for example. We also provide that as output of our process.

As an example, we have computed the volume and the surface of a sphere in a structured hexahedral mesh. A first order integration was performed to obtain both the volume of the elements inside the sphere and the area of the triangles on the surface of the sphere (see figure below).



Secondly, we simulated the behavior of a composite material consisting in a matrix in wich spherical inclusions are embedded, under uniaxial tensile test.

## REFERENCES

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