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LOW-ORDER MODELS BASED ON 2D AND 3D NAVIER-STOKES SIMULATIONS OF COMPRESSIBLE UNSTEADY FLOWS

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ABSTRACT

Reduced-Order Models (ROM) of the unsteady Navier-Stokes equations are developed in the perspective of integrating compressible flow computations into iterative processes like optimal shape design or fluid-structure interaction. A strong reduction of the number of degrees of freedom is reached by a truncated Proper Orthogonal Decomposition (POD) of the state variables. The POD enables an efficient extraction of the flow main dynamics. A Galerkin projection of the “high-fidelity” physical model onto the POD basis is performed leading to a low-dimensional ordinary differential equation system. Many ROM of the Navier-Stokes system have been developed under the incompressibility assumption. Two difficulties appear in the compressible context because of the coupling of kinematic and thermodynamic state variables. A specific formulation concerning density and pressure [1] has to be considered in order to maintain quadratic fluxes in Navier-Stokes equations as in the incompressible case. Furthermore, the definition of the inner product involved in the POD has to be reconsidered. In the present study, a consistent inner product is suggested to derive ROM of the fully compressible Navier-Stokes equations. The “high-fidelity” Navier-Stokes simulations are issued from ICARE/IMFT finite volume flow solver.

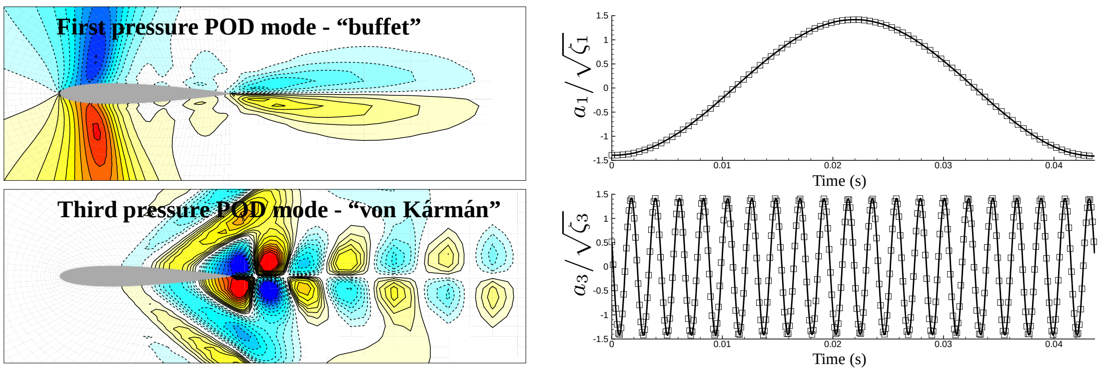

Figure 1: *Identification of flow unsteadiness by pressure POD modes and associated temporal dynamics: projection of Navier-Stokes simulation snapshots (reference, ☐) and ROM prediction (plain lines).*

As a first step, this model reduction technique has been validated on two-dimensional transonic flows ($M_a \in [0.8, 0.85]$) around a NACA0012 airfoil at zero angle of incidence and moderate Reynolds numbers ($R_e \in [0.5, 1] \times 10^4$) [2]. The POD leads to a precise identification of flow unsteadiness induced by compressibility effects: von Kármán instability and buffet phenomenon (Fig. 1).

POD mode temporal dynamics are the "state variables" of the low-dimensional model which have te be predicted by the ROM. ODE systems issued from POD-Galerkin methodology are structurally unstable which induces strong phase drifts for long time integrations. This instability is controlled a posteriori by a linearised calibration procedure of very low computaional cost. Calibrated ROM lead to faithful predictions of the reference dynamics (Fig. 1). ROM yields an accurate prediction of the aerodynamic coefficients (Fig. 2). The robustness of the ROM towards changes in flow parameters has been studied in the case of Reynolds number variations and different strategies have been put forward to include these changes in the ROM: control function, continuous mode interpolation... The model reduction technique is applied to a more chaotic flow: the transitional three-dimensional flow around a NACA0012 wing at 20 degrees of incidence ($R_e = 800$, $M_a = 0.3$) (Fig. 3) and ROM predictive capacities are examined.

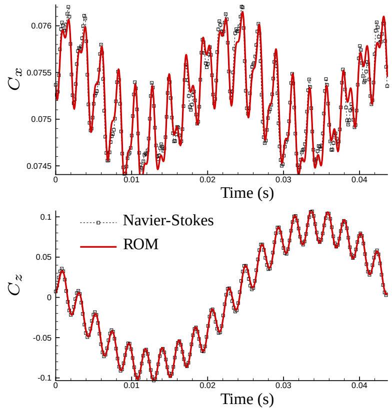

Figure 2: *Aerodynamic coefficient prediction by Navier-Stokes simulation (□) and ROM (plain line), over a period of buffeting.*

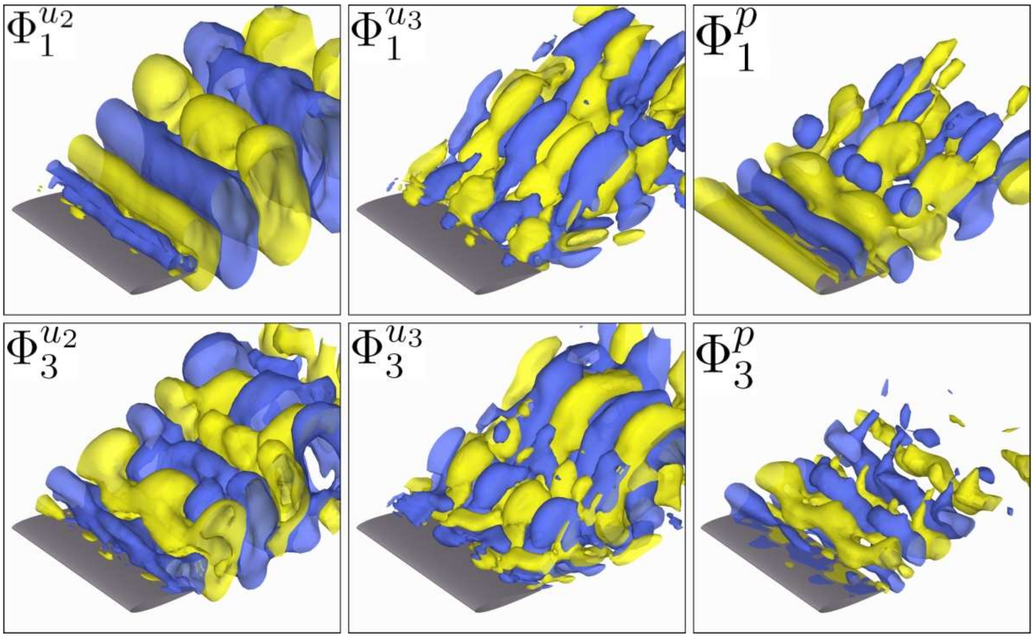

Figure 3: *Secondary instability in the flow around a NACA0012 wing: first and third POD modes associated to the vertical and transversal velocity components (left and center) and to the pressure (right).*

REFERENCES

[1] G. Vigo, A. Dervieux, M. Mallet, M. Ravachol and B. Stoufflet. "Extension of methods based on the proper orthogonal decomposition to the simulation of unsteady compressible Navier-Stokes flows". *Computational Fluid Dynamics'98, Proc. of the Fourth ECCOMAS Conf.*,, 648–653, Wiley, 1998.

[2] R. Bourguet, M. Braza and A. Dervieux. "Reduced-order modeling for unsteady transonic flows around an airfoil, *Phys. Fluids, Vol.* **19**, *111701:1–4, 2007.*