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Numerical continuation methods for models with nonsmooth dynamics

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ABSTRACT

Many problems in mechanics involve nonsmooth events such as impacts or switching. For example the classic problem of intermeshing gear teeth: two gears, A and B, can be in one of three modes of operation, A driving B, B driving A, and freeplay, where A and B are not in contact. When viewed as a dynamical system, the gears define a piecewise smooth vector field with nonsmooth transitions between the different modes of operation.

While for analysing smooth dynamical systems there are many computational tools available, there are very few tools for analysing nonsmooth dynamical systems beyond numerical integration. Powerful tools such as numerical continuation, where a solution of the system can followed through parameter space and the bifurcations it undergoes are tracked, are not widely available for nonsmooth dynamical systems.

Here, we present a new method for the numerical continuation of nonsmooth periodic orbits in piecewise smooth dynamical systems with delay. This method allows detailed studies to be performed to investigate the dynamics of a system near to, or after, a nonsmooth bifurcation, such as a grazing point. This enables one to investigate the dynamics caused by, for example, impacting behaviour in a system.

The method we present is built around a multi-point boundary value solver for delay differential equations. This approach allows us to consider arbitrarily many nonsmooth events per period and arbitrary conditions for switching between vector fields can be imposed. Furthermore, it is straight forward to incorperate effects such as sliding, where the motion of the system follows the discontinuity boundary between two different vector fields.

To implement the multi-point boundary value solver we use the method of collocation with orthogonal polynomials. Collocation is a very general method, used in a wide variety of computational software packages, that approximates a solution using piecewise polynomials. It has been found to be a very robust method and produces highly accurate solutions.

To illustrate this method, we focus on a 2 degree of freedom model of the turning cutting process that incorperates contact losses. The model is formulated as a piecewise smooth delay differential equation

with a state-dependent delay. The delay arises from the time taken for a single revolution of the work piece (the cutting force is dependent on the difference between the current position of the cutting tool and its position one revolution earlier), and it is state dependent since we allow for vibrations in the direction of rotation.

Contact losses occur in the model after the trivial steady cutting state loses stability at a Hopf bifurcation. The periodic orbits that emerge from the Hopf bifurcation grow in amplitude until they exceed the cutting depth, at which point they lose contact with the work surface. The moment cutting tool loses contact from the work piece there are two nonsmooth events per period. Firstly, the cutting force goes to zero over a finite interval of time and, secondly, there is a discontinuous change in the time delay at two points over the period. Thus, the initial contact loss can induce very complicated dynamics.

The complicated dynamics seen at the contact loss correspond to the phenomenon known as machine tool chatter, where the cutting tool bounces chaotically on the surface of the work piece and creates a badly formed cutting surface. We shall show how the chattering behaviour changes as the system parameters change. In particular, we will show the change in behaviour as the initial destabilising Hopf bifurcation changes from subcritical to supercritical.