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Reliability analysis of shells based on direct plasticity methods

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ABSTRACT

Practical design codes often prescribe the values of safety factor of the resistance and of the loads for a given problem since these variables are generally uncertain. Recent advance in computer technology have opened up a way to consider uncertainties in structural parameters such as loads, geometry and material properties. Reliability has been introduced into design practice as one of the most important and objective measure of the quality and performance of a structure.

This paper concerns the application of a new algorithm of probabilistic limit and shakedown analysis for shell structures, in which the loading and strength of the material as well as the thickness of the shell are considered as random variables. The procedure involves a deterministic limit and shakedown analysis for each probabilistic iteration, which is based on the kinematical approach and the use of the re-parameterized exact Ilyushin yield surface proposed by Burgoyne and Brennan. By imposing a consistent relationship between the velocity fields and the kinematically admissible strain rate fields, the shakedown or limit analysis problems can be reduced to a non-linear minimization problem and solved by Newton's method in conjunction with a penalty method and the Lagrange dual method. A special "smooth regularization method" is also used for overcoming the non-differentiability of the objective function. The actual Newton directions are updated at each iteration by solving a purely-elastic-like system of linear equations which ensures automatically the kinematical condition of the displacements.

The limit state function separating the safe and failure regions is defined directly as the difference between the obtained limit load factor and the current load factor. Different kinds of distributions of the basic variables are taken into consideration and the failure probability of the structure is calculated with First and Second Order Reliability Methods (FORM/SORM). A non-linear optimization was implemented for finding the design point, which is based on the Sequential Quadratic Programming. Non-linear sensitivity analyses are also performed for computing the Jacobian and the Hessian of the limit state function.

The advantage of the method is that the failure under cyclic loading is treated as a time-invariant problem and only few material data is required. Moreover, sensitivity analyses are obtained directly from the solution of the shakedown problem with no extra computational cost. Numerical examples are tested against literature with analytical method and with a numerical method using volume elements. The proposed method

appears to be a robust method of identifying good estimates of the failure probability, even in the case of incomplete data and very small probabilities. Since the limit state function is linear with the limit and shakedown analysis, then the use of FORM is sufficient and SORM is only necessary if the linearity is lost after the transformation into the standard normal space.

A numerical example

A pipe bend is investigated with the random variables in-plane bending moment M_I, yield stress σ_y and wall thickness h, with mean values μ_s, μ_r, μ_t and standard deviations σ_s, σ_r, σ_t respectively. The failure probabilities P_f are calculated for different distributions of random variables. Numerical solutions of the limit and shakedown analyses compare well with the exact analytical solutions.

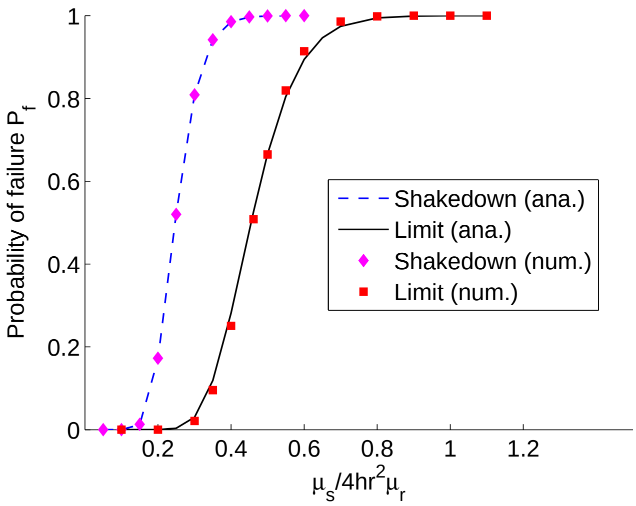

Fig.1. P_f for log-normally distributed variables, $\sigma_r = 0.2\mu_r$, $\sigma_s = 0.1\mu_s$.

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