8th. World Congress on Computational Mechanics (WCCM8)
5th European Congress on Computational Methods in Applied Sciences and Engineeering (ECCOMAS 2008)
June 30 –July 5, 2008
Venice, Italy

Numerical investigation of finite element discretizations for hyperelastic materials in large deformations

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Key Words: *Finite element discretizations, Iterative solvers, hyperelastic material.*

ABSTRACT

Solid mechanics problems involving non linear materials and large deformations are still challenging. Various numerical methods can be used to solve them but in all cases, whatever the method, the implementation is complex and the possibility of erroneous code remains present. These errors can take various forms from the basic bug (detected or not at compilation) to wrong convergence behaviour due to an erroneous tangential matrix in a Newton method for instance. It is also important to clearly establish the appropriate convergence properties of the discretizations with mesh size. It should be clearly established that second order discretizations give second order convergence with mesh size. When all these verications have been performed, we can be confident that our numerical code is correct (though some small errors almost always remain).

The classical way to make sure that a numerical method is correctly implemented is to solve problems where an analytical solution is known. These problems are however scarce and in many instances, they are oversimplified so that solving them is not completely conclusive. Another way consists in solving benchmark problems and to make comparisons with the existing literature. This is certainly a valid approach but the comparisons are often only qualitative and therefore incomplete. Moreover, the comparisons are not always possible when for example, one wants to test a new formulation, a new model or a new constitutive equation.

Finally, the question of the discretizations of the PDE system is still debated. A vast choice of possibilities exists ranging from displacement only to various mixed (displacement and pressure) formulations using hexahedrals or tetrahedrals in dimension 3, triangles or quadrilaterals in dimension 2. The computational performance of these discretizations is also debated. Low order hexahedral (bilinear) elements are often favoured since it is believed that they are more cost effective. A first objective of this presentation is to demontrate that it is not the case and to present a very efficient iterative method specially designed for quadratic discretizations based on a hierarchical basis (see El Maliki [1]).

It is also still believed that tetrahedral elements are inadequate for incompressible material problems. A second objective of this presentation to propose various hexahedral and tetrahedral discretizations,

and to show that they essentially have the same convergence properties. Tetrahedral elements are therefore favoured since complex geometries are more easily meshed with tetrahedrals and moreover, most adaptive remeshing strategies are based on tetrahedrals.

Finally, we will propose a more efficient way (see Chamberland et al. [2]) to verify all these properties based on the concept of so-called manufactured solutions. A manufactured solution is simply an analytical solution to a slightly modified version of the PDE system under consideration. The modification consists in general in the addition of an artificial source term which is easily implemented.

References

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