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ACCELERATING AN ELASTODYNAMIC BOUNDARY ELEMENT FORMULATION BY USING ADAPTIVE CROSS APPROXIMATION

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ABSTRACT

The understanding and modelling of elastodynamic phenomena are of great importance in engineering applications. Especially, the simulation of dynamic effects in half-spaces are of interest, e.g., in earthquake engineering. For the calculations in such unbounded domains the Boundary Element Method (BEM) is the preferred methodology. For elastodynamics this method has been introduced by Cruse and Rizzo [3, 2].

This numerical technique has reached meanwhile a state of maturity also for dynamic problems. For time dependent problems a step forward has been done due to the work of Lubich [5, 6] and Schanz [8]. However, a dynamic calculation is much more time and storage consuming compared to static calculations and for the classical BEM the matrices are also fully populated. Hence, the numerical as well the storage complexity for establishing the system matrices is asymptotically of quadratic order with respect to the unknowns. This restricts a solvable problem to a rather small or probably medium size.

To overcome this restriction several methodologies have been developed to reduce the memory requirement as well as the calculation time [7]. The main idea is an approximation of the system matrices by compression techniques, i.e., to employ data sparse techniques.

Several approaches, such as the fast multipole method or the panel clustering, gain their efficiency basically from an analytic approximation of the non-local kernel function. This can be achieved by the decomposition of the kernel function

$$\mathbf{K}(\mathbf{x}-\mathbf{y}) \approx \bar{\mathbf{K}}(\mathbf{x},\mathbf{y}) = \sum_{i=1}^{r} \mathbf{p}_i(\mathbf{x})\mathbf{q}_i(\mathbf{y}) \ ,$$

into a product of functions, one dependent on the field point $\mathbf{x}$ and one dependent on the load point $\mathbf{y}$. Thus, the dependency on the distance between the two points is approximated. The main difficulty is to find a suitable series expansion for the explicit separation of the kernel function. This is not yet solved for elastodynamics.

The present work focuses on a purely algebraic approach, i.e., the Adaptive Cross Approximation (ACA) as proposed in [1] is applied. In contrast to the analytic approaches as sketched above, this technique is not explicitly dealing with the kernel functions. Only the geometric information of the

problem under consideration is necessary. The main idea is that the product of r pivot vector pairs $\mathsf{u}\mathsf{v}^\top$ build up a low rank matrix $\tilde{\mathsf{A}}$ with rank r

$$\tilde{\mathsf{A}} = \sum_{i=0}^{r} \mathsf{u}_i \mathsf{v}_i^\top \ .$$

Iteratively this product is upgraded by an additional pivot cross, until in the r^{th} iteration step a defined accuracy of the low rank matrix is reached, i.e., until

$$\|\mathsf{A} - \tilde{\mathsf{A}}\|_F \leq \epsilon \|\mathsf{A}\|_F$$

holds. Hence, the advantage of the ACA is based on the fact that only a few of the original matrix entries have to be generated and stored. The residual entries can be neglected. This represents the physical effect that the influence of two points with a large distance is much smaller than that of two neighbouring points.

In the numerical procedure, first, a so called $\mathcal{H}$-matrix [4] has to be set up. Therefore, the original system matrix is subdivided into admissible near- and far-field blocks by means of a geometrical clustering. Then each far-field block can be represented by the above mentioned adaptive low-rank approximation.

As will be shown numerically, the presented approach is suitable for the efficient simulation of elasto-dynamic problems.

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