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On the Reliability of Krylov Subspace Methods and Alternatives

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ABSTRACT

The need for solving linear systems of equations permeates many areas of computational science and engineering such as computational mechanics (structures, fluids, and fluid-structure interaction), electronic circuit simulation and electromagnetics. The linear systems that arise in these practical applications are almost always large and sparse with the coefficient matrices either available explicitly or via a matrix-vector multiplication kernel. Resorting only to direct sparse solvers is often not possible due to the large size of these systems or due to the cost of assembling the matrices explicitly. The family of Krylov Subspace methods that includes the Conjugate Gradient scheme for the normal equations (CGNR), Generalized Minimum Residual (GMRES), and the stabilized Bi-Conjugate Gradient (BiCGstab) algorithms, is often used to handle those nonsymmetric linear systems with or without a preconditioning strategy. With the availability of large hierarchical memories in recent parallel computing platforms, we will assume in this paper that the linear system is explicitly available.

Preconditioning is almost always needed for the success of these methods. In the absence of intimate knowledge of the application at hand, users often resort to the use of ”black-box” preconditioners such as Incomplete LU Factorization (ILU), Sparse Approximate Inverse (SPAI), as well as approximate orthogonal factorization. In this paper, we consider the improved most recent version of the ILUT preconditioner. This version of ILUT adopts preprocessing steps to achieve the following: (i) reordering to bring the largest elements in magnitude to the diagonal, (ii) reordering to minimize the bandwidth, and (iii) scaling of the diagonal entries to ensure that the matrix is balanced [1].

The reliability of the above Krylov subspace schemes across different types of linear systems has received little attention, e.g. see [3]. Using the University of Florida Matrix Collection [2], which contains a set of large linear systems generated from a variety of real-world applications, we explored the rates of success of CGNR, full GMRES, and BiCGstab on the 65 largest nonsymmetric systems of this collection [4]. Using the above most recent version of the ILUT preconditioner, the rates of success of these three schemes are 11%, 34% and 34%, respectively. It is of interest to note that a single method is not consistently better or worse than another, being impossible to predict a priori if it will fail or succeed on a given problem.

The development of more robust methods is essential if one is to handle systems across different applications or even across different aspects of a single application. These alternatives must be more reliable, and should allow solving systems with multiple right-hand-sides, and guarantee scalability on massively parallel computing platforms. These include accelerated Block Row Projection schemes, Chebyshev-based Iterations, and eigenvalue problem-based approaches. In this paper we present some of these alternatives, compare them with the above Krylov subspace family, and present their performance on few selected parallel architectures.

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