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Energy norm a-posteriori error estimation for hp-adaptive DG methods for convection-diffusion equations

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ABSTRACT

We develop the energy norm a-posteriori error estimation of hp-adaptive discontinuous Galerkin (DG) finite element methods for stationary convection-diffusion equations.

We begin by discussing hp-version error estimation for second-order elliptic problems. Following the recent work [1], we derive computable upper and lower bounds on the error measured in terms of a natural (mesh-dependent) energy norm. The bounds are fully explicit in the local mesh sizes and approximation orders. We present a series of numerical experiments illustrating the performance of the estimators in an automatic hp-adaptive refinement strategy and show that the resulting adaptive scheme are able to resolve solution singularities at exponential rates of convergence.

We then apply our techniques to derive energy norm error estimators for convection-diffusion equations and present a new estimator for the error measured in terms of the energy norm and a dual norm associated with the convective terms in the equations, see [2]. It is shown that the ratio of the upper and lower bounds is independent of the magnitude of the Péclet number of the problem, and hence the estimator is robust for convection-dominated problems. This is illustrated in a set of numerical tests. These tests also indicates that, in many instances, that dual norm in the error measure can be neglected and the estimators are robust with respect to the energy norm alone.

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