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FRACTIONAL NUMERICAL METHODS IN GEOTECHNICS

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ABSTRACT

Although the fractional calculus appeared almost than the differential and integral calculus, recently (since 1980) it has been received greater interest from investigators of different branches of the science and engineering. This is due because the fractional derivatives have shown to be representative tools of natural phenomena, by other way, for to study the mechanical behavior of granular media, as well as other types of phenomena (economic, biological, etc). In fact, in the case of granular media, this is of great interest in geotechnics, so this discipline is powered incorporating to its methodologies this branch of the modern mathematics.

Therefore, among the objectives of this article, as well as in the investigation line of the authors, there are the following: Firstly, the use of the fractional calculus and its numerical methods in diffusion problems (flow of water in soils), wave propagation (earthquakes) and the formulation of constitutive models (rheological). Another objective, is to compare the results obtained (of this kind of problems) with the obtained by classical methods (derivatives of entire order). In this article, concepts of fractional calculus [1], fractional differential equations, numerical methods [2] and examples of application in geotechnics are presented.

The numerical solution for the diffusion equation using finite differences with a scheme of fractional differences [3], for the two-dimensional case (plate) is given by the equation (1). A computer program was elaborated for this model.

$$U_{i,j}^{(m+1)} = U_{i,j}^{(m)} + Sx\sum_{k=0}^{m} \omega_k^{(1-\gamma)} \left[U_{i,j-1}^{(m-k)} - 4U_{i,j}^{(m-k)} + U_{i,j+1}^{(m-k)} + U_{i-1,j}^{(m-k)} + U_{i+1,j}^{(m-k)}\right] \quad (1)$$

Two series of analysis of diffusion heat in a rectangular plate with homogeneous properties were carried out. The first analysis series was carried out varying the order of fractional derivation (gamma) and maintaining constant the diffusion coefficient. The initial conditions of temperature for this series were: assigning a lineal variation of temperatures in the left vertical frontier and flow condition with null derivative in the right vertical frontier. For the second series the initial conditions were: In the left inferior corner a punctual source of cooling was placed. Two analyses were made: a)

varying the coefficient, maintaining constant the value of order gamma and b) varying the fractional order gamma, maintaining constant the coefficient. Diminishing the coefficient or the order the diffusion is slower. In the figures 1 and 2 the distribution of temperatures is presented for the step 2 and for the step 40) for the fractional order 0.6, for the other cases the geometric distribution is very similar, it only varies the evolution on the time. If one observes the diffusion for some points of the plate through the time, it is seen that differences exist in the evolution of the temperature abatement in each of them.

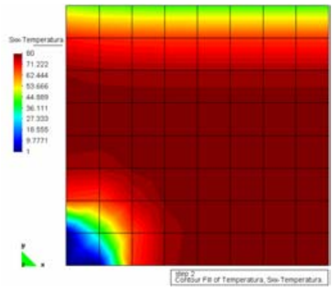

Figure 1:Step 2

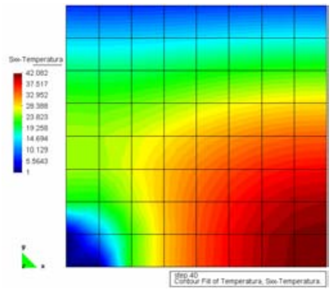

Figure 2. Step 40

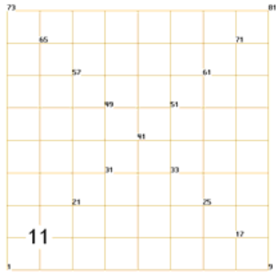

Figure 3. Grid Numeration

In the figure 4 the graphs of abatement are presented in each one of the nodes in the diagonal 1-81 (figure 3). In the figure 5 the corresponding graphs are presented for the diagonal 9-73 (figure 3). In both cases the results for t=4.0 are presented.

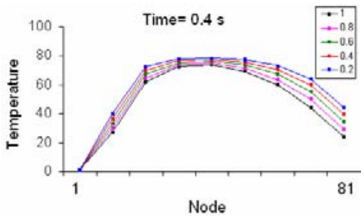

Figure 4. Diagonal 1-81

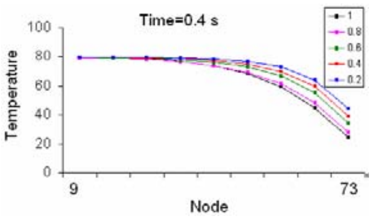

Figure 5. Diagonal 9-73

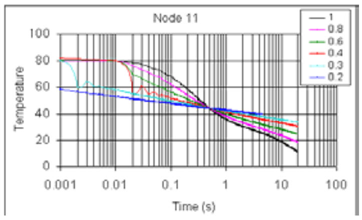

Figure 6. Abatement node 11

In the figure 6 the graph of T vs log(t) is presented for one of the nodes in the plate. A strong fall of temperature can be noticed (for gamma <=0. 4) before the corresponding time for the intersection point of all the curves. That time passing the loss of temperature is smaller for gamma <=0. 4 (that is to say the phenomenon is slower). Among the conclusions we have: a) there are different abatements values in the distinct points of the plate, and b) the fractional diffusion equation (order 0-1), it can be useful to model the behavior of partially saturated soils. Also recommendations for studies with derivation order between one and two are given, and also for experimental works with partially saturated soils, to use the fractional models seen in this work.

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