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IMPROVED SIF EXTRACTION USING DIFFERENT DOMAIN INTEGRALS IN FEM AND X-FEM

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ABSTRACT

In recent years, the SIF extraction using domain integrals as a post-processing technique of a FEM or X-FEM solution has become customary and has clearly overcome prior techniques based on local extrapolation. The equivalent domain integral to the contour integral J was proposed in [1]. For the analysis of mixed mode LEFM problems, the recasting of the I integral as a equivalent domain integral is commonly used. The I integral was proposed by Chen *et al.* [2] and introduces the asymptotic LEFM fields as auxiliary or extraction fields [3].

In this work, we study the extraction of the mode I stress intensity factor in 2D LEFM problems using FEM and X-FEM by means of these two widespread domain integrals. By means of numerical examples, it is shown that the ratio between the errors committed when using the domain forms of the J and I integrals is approximately constant for a given problem. This enables the computation of a much more accurate SIF by combining the results obtained via J and I using two different meshes. In general, the exact error in the SIF K_{I}^{J} (computed through the domain integral J) and the exact error in K_{I}^{I} (computed through the domain integral I) can be defined as:

$$e(K_{\mathrm{I}}^{J}) = K_{\mathrm{I}}^{\mathrm{ex}} - K_{\mathrm{I}}^{J} \quad ; \quad e(K_{\mathrm{I}}^{I}) = K_{\mathrm{I}}^{\mathrm{ex}} - K_{\mathrm{I}}^{I} \tag{1}$$

In this work, these errors have been computed for reference problems with exact solution and it has been verified that the ratio between both errors tends to be approximately constant for a given problem. This constant R is defined as:

$$R = \frac{e(K_{\mathrm{I}}^{J})}{e(K_{\mathrm{I}}^{I})} = \frac{K_{\mathrm{I}}^{\mathrm{ex}} - K_{\mathrm{I}}^{J}}{K_{\mathrm{I}}^{\mathrm{ex}} - K_{\mathrm{I}}^{I}} \tag{2}$$

The constant R depends on the problem analyzed and, in general, $R \neq 1$. It reflects the fact that the discretization error inherent in a finite element solution $\mathbf{u}^{\mathrm{fe}}$ affects in a different way to the SIF computation through the domain integrals J or I. For a given problem, it has been observed that R does not depend on the discretization used. This implies that the errors $e(K_{\mathrm{I}}^{J})$ and $e(K_{\mathrm{I}}^{I})$ exhibit the same convergence rate and the same convergence pattern when the discretization is changed varying the element size h or the interpolation order p.

The proposed methodology has some advantages over the Richardson's extrapolation. When using Richardson's extrapolation, the convergence rate must be known *a priori* (or, at least, it must be computed). Besides, it can only be applied if the error converges monotonically [4]. The improvement proposed in this work eliminates the necessity of knowing the convergence rate, the element size or the number of degrees of freedom.

Several numerical mode I examples have been solved both using FEM and X-FEM, verifying that the constant R is approximately constant for each problem and independent of the discretization. This has enabled to obtain an improved SIF estimation based on the values K_I^J and K_I^I (obtained through J and I respectively) for any two different meshes without an *a priori* knowledge of the convergence rate. This is exemplified in Figure 1.

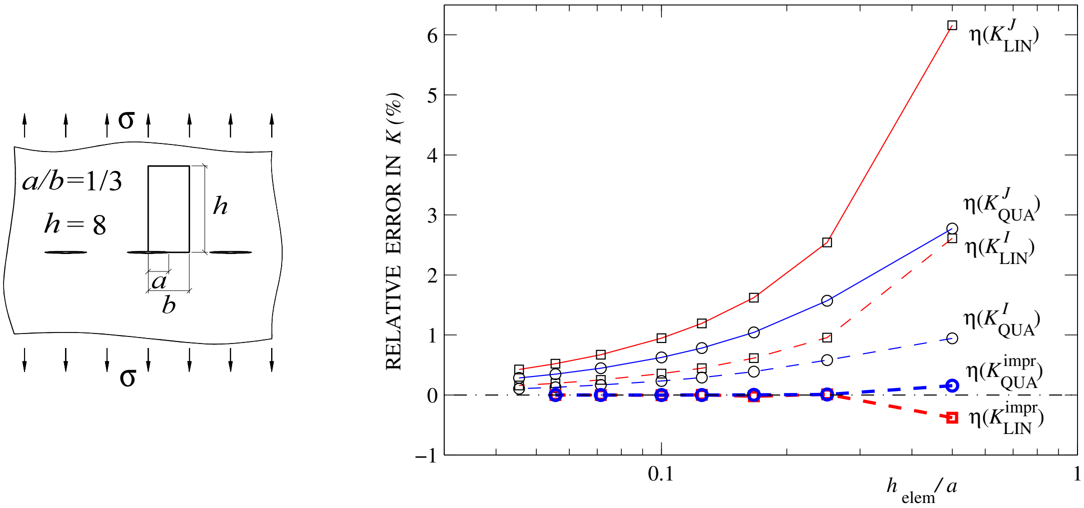

Figure 1: Left: Reference problem. Right: Improved estimation of K for a sequence of meshes.

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